MATH 4P85 SECOND MIDTERM MARCH 16, 2011

One sheet of notes and a Maple workspace (loaded from a memory stick) containing any information are allowed.

Full credit given for correctly answering 3 (out of 5) questions.

Duration: 50 minutes

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1. Consider a LGWI process with the following rates:

$$\begin{aligned}\lambda_n &= 7.04n + 2.34 \text{ per hour} \\ \mu_n &= 8.12n \text{ per hour}\end{aligned}$$

and consisting of 11 'members' at 8:38. Compute

(a) the expected value of the process at 9:31, and the corresponding standard deviation,

(b) probability that, at 9:31, the process has more than 13 members,

(c) the expected value and standard deviation of the time of 'death' of the last of the initial 11 members (express in hours : minutes : seconds),

(d) the expected value and standard deviation of the time of 'death' of the last 'native' (the native sub-population consists of the 11 initial members and all their descendents).

2. This is a continuation of the previous question. Compute:

(a) the expected number of (surviving) immigrants (do *not* include any of their descendents) at 9:31, and the corresponding standard deviation,

(b) the expected number of (surviving) immigrants *and* their descendents at 9:31, and the corresponding standard deviation,

(c) the percentage of time (in the long run) with no surviving immigrants (ignore their descendents),

(d) the percentage of time (in the long run) with no surviving immigrants nor their descendents (this is what we call State 0).

3. Continuation: Compute the probability

   (a) of at least 5 immigrants arriving by 9:31,
   (b) that all immigrants arriving before 9:31 will die ‘childless’ (without any offsprings),
   (c) that, at 9:31, the immigrant and native sub-populations are equal in numbers.

4. Find the general solution to

$$(z\cos z - \sin z)\,\dot{P}(z,t) = z^2 P'(z,t)$$

   Also, find the specific solution which meets

$$P(z,0) = \frac{z^2}{1-\cos^2 z}$$

5. Find the general solution to

$$(1-z)\dot{P}(z,t) = z^2 e^{-z} P'(z,t) + (1-z)P(z,t)$$

   (assume that $0 \le z < 1$). Also, find the specific solution which meets

$$P(z,0) = 1$$