MATH 4P85 SECOND MIDTERM MARCH 22, 2012

One sheet of notes and a Maple workspace (loaded from a memory stick) containing any information are allowed. Full credit given for correctly answering 3 (out of 5) questions. **Duration: 50 minutes**

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1. Consider a LGWI process with the following rates:

$$\begin{aligned} \lambda_n &= 8n + 2 \text{ per hour} \\ \mu_n &= 8n \text{ per hour} \end{aligned}$$

and consisting of 10 'members' at 8:15. Compute:

(a) the expected value of the process at 8:42, and the corresponding standard deviation,

(b) the probability that, at 8:42, the process has more than 12 members,

(c) the probability that the last 'native' (those are the initial 10 members and their progeny) dies between 10:00 and noon,

(d) the probability that the last one of the initial 10 members dies between 8:30 and 8:45.

2. Find the general solution to

$$\dot{P}(z,t) = (e^z - 1) \cdot P'(z,t)$$

(assume that $0 \le z < 1$). Also, find the specific solution which meets

$$P(z,0) = \ln(1 - e^{-z})$$

3. Consider a B&D process with the following rates

$$\begin{aligned} \lambda_n &= 7.2 \text{ per hour} \\ \mu_n &= 1.1 \times n \text{ per hour} \end{aligned}$$

and the value of 10 at 8:15. Compute

(a) the expected value of the process at 8:42, and the corresponding standard deviation,

(b) the probability that, at 8:42, the process has more than 12 members,

(c) the probability that the last one of the initial 10 ‘members’ leaves the system (‘dies’) between 10:00 and noon.

4. Continuation of the previous question (the corresponding process can be clearly identified with an $M/M/\infty$ queue). Using the same initial condition (10 customers at 8:15), compute:

   (a) the probability of at least 3 new arrivals before *any* customer completes his/her service (hint: think in terms of the next three *transitions* - they must all be ’up’),

   (b) the expected number (and the corresponding standard deviation) of customers who arrive between 8:15 and the time of departure of the last one of the 10 people currently in service (hint: Poisson process of random duration),

   (c) the long-run frequency of visits to State 0 (per week), and their average duration (in minutes and seconds).

5. Find the general solution to

$$\dot{P}(z,t) = (e^z + 1) \cdot P'(z,t) + (e^z - 1) \cdot P(z,t)$$

(assume that $0 \le z < 1$). Also, find the specific solution which meets

$$P(z,0) = \frac{1}{e^z + 1}$$