MATH 4P85 SECOND MIDTERM MARCH 19, 2013

Open book exam.
Full credit given for correctly answering 3 (out of 5) questions.

Duration: 50 minutes

---

1. Consider a LGWI process with the following rates:

$$\begin{aligned} \lambda_n &= 32 + 63 \times n \text{ per day} \\ \mu_n &= 74 \times n \text{ per day} \end{aligned}$$

(a day means 24 hours - this is a round-the-clock operation) and having 9 'members' at 8:12. Compute

   (a) probability that, at 9:51, the process has between 4 and 10 members (inclusive),

   (b) the expected value and standard deviation of the time of 'death' of the last 'native' (the native sub-population consists of the 9 initial members and their progeny),

   (c) the expected number of (surviving) 'immigrants' (do *not* include any of their descendents) at 9:51, and the corresponding standard deviation,

2. This is a continuation of the previous question. Compute:

   (a) the expected number of (surviving) immigrants *and* their (surviving) descendants at 9:51, and the corresponding standard deviation,

   (b) the percentage of time (in the long run) with no surviving immigrants (ignore their descendents),

   (c) the probability that all immigrants arriving before 9:51 will die 'childless' (without any offspring).

3. Find the general solution to

$$(1 + \ln z) \cdot \dot{P}(z,t) = P'(z,t)$$

Also, find the specific solution which meets

$$P(z,0) = z^z$$

4. Find the general solution to

$$\left(\dot{P}(z,t) - P(z,t)\right) \cdot \cos z = P'(z,t) \cdot \sin z$$

Also, find the specific solution which meets

$$P(z,0) = 1$$

5. Consider a B&D process with the following rates

$$\begin{aligned} \lambda_n &= 7.2 - 0.6 \times n \text{ per minute} \\ \mu_n &= 0.9 \times n \text{ per minute} \end{aligned}$$

and the value of 6 at 8:39:12. Compute

(a) the probability that the next three transitions are all 'births',

(b) the expected value of the process at 8:42:06, and the corresponding standard deviation,

(c) the long-run frequency of visits to State 12 (per hour), and their average duration (in seconds).