

Open book exam.

**Duration: 50 minutes**

Full credit given for correctly answering 6 (out of 10) questions.

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1. Consider a LGWI process with the following rates

$$\lambda_n = 25 + 6.2n \text{ per hour}$$

$$\mu_n = 7.3n \text{ per hour}$$

having 8 members at 8:17. Compute

- the expected value (use the xx:yy:zz format) and standard deviation (in minutes and seconds) of the time of death of the last native ('natives' are the 8 initial members *and* their progeny),
  - the long-run proportion of time with more than 5 (surviving) immigrants (this time, do *not* include their descendents).
2. This is a continuation of the previous question. Compute:
- the probability that the *total* (combining natives, immigrants and their progeny) surviving population at 8:55 is between 12 and 17,
  - the expected value of this population at 8:55, and the corresponding standard deviation.

3. Consider the following PDE

$$\dot{P}(z, t) = P'(z, t) \cdot (1 + z^2) \cdot \arctan z$$

- Find its general solution (sufficiently simplified). Give details of your steps, not just the final answer.
- Also, find the specific solution which meets

$$P(z, 0) = \frac{1}{1 + z^2}$$

Spell out your  $g(x)$  function - what exactly is it?

4. Consider the following PDE

$$\dot{P}(z, t) = (1 + e^{-z}) \cdot P'(z, t) + P(z, t)$$

- (a) Find its general solution (simplified). Provide details.
- (b) Also, find the specific solution which meets

$$P(z, 0) = 1 + e^z$$

What was your  $g(x)$ ?

5. Consider a B&D process with the following rates

$$\lambda_n = 7.5 - 0.5n \text{ per hour}$$

$$\mu_n = 0.8n \text{ per hour}$$

and the value of 10 at time 0. Compute

- (a) the expected value of the process 18 minutes later, and the corresponding standard deviation,
- (b) the long-run average frequency of visits to State 12 (per week, assuming 24/7 operation), and their average duration (in minutes and seconds).