MATH 4P85          SECOND MIDTERM          MARCH 19, 2015

Open book exam, use of Maple allowed.          **Duration: 1 hour**
Full credit given for correctly answering 6 (out of 10) questions.

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1. Consider a LGWI process with the following rates

$$\lambda_n = 11 + 5.2n \quad \text{per hour}$$
$$\mu_n = 6.3n \quad \text{per hour}$$

having 9 members at 8:24:17. Compute

(a) the expected value (use the xx:yy:zz format) and standard deviation (in minutes and seconds) of the time of extinction of the native sub-population (initial members and their progeny),

(b) the long-run proportion of time with more then 6 members of the immigrant sub-population (immigrants and their progeny).

2. Consider an $M/M/\infty$ queue with customers arriving at the rate of 15.3 per hour, the expected service time of 12 minutes and 5 seconds, and 7 customers being serviced at 9:22. Compute

(a) the expected number of customers being serviced at 9:37 and the corresponding standard deviation,

(b) the long-run frequency of visits to State 0 (per 24 hour day) and their average duration (in minutes and seconds).

3. Find (show the individual steps)

(a) the general solution to

$$e^{2z} \cdot \dot{P}(z, t) + P'(z, t) = 0$$

(b) and the specific solution which also meets (spell out what your $g(x)$ is)
$$P(z, 0) = \sin(1 - 3 \cdot e^{2z})$$

4. Find (show the individual steps)

(a) the general solution to

$$e^{2z} \cdot \dot{P}(z,t) + P'(z,t) + 2P(z,t) = 0$$

(b) and the specific solution which also meets (spell out what your $g(x)$ is)

$$P(z,0) = \sin(1 - 3 \cdot e^{2z})$$

5. Consider a B&D process with the following rates

$$
\begin{aligned}
\lambda_n &= 26 - 2n \quad \text{per minute} \\
\mu_n &= 1.7n \quad \text{per minute}
\end{aligned}
$$

and the initial value of 7. Compute

(a) the probability that the process will have more than 9 members 16 seconds later.

(b) the long-run frequency of visits to State 5 and their average duration.