MATH 4P85 SECOND MIDTERM MARCH 21, 2016

Open book exam. Students are allowed to use Maple to answer the first 3 (but not the last 2) questions. Full credit given for correctly answering 6 (out of 10) sub-questions. **Duration: 1 hour**

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1. Consider a LGWI process with the following rates:

$$\begin{aligned}\lambda_n &= 5.4n + 9.8 \text{ per hour}\\ \mu_n &= 7.2n \text{ per hour}\end{aligned}$$

and consisting of 11 members at 8:14. Compute

(a) the probability that, at 8:38, the process has more than 14 members,

(b) the expected number of surviving 'immigrants' (*not* counting their descendents) at 8:38, and the corresponding standard deviation,

(c) the expected time (use the xx:yy:zz format) of death of the last 'native' (these are the initial members *and* their descendents) and the corresponding standard deviation,

(d) the long-run proportion of time during which the process has a value bigger than 14.

2. Consider the $M/M/7$ queue with the average arrival rate of 107.2 customers per hour, and the average service time equal to 2 minutes and 37 seconds. Find the long-run

(a) proportion of time with all 7 servers idle,

(b) proportion of time each server spends idling (assuming the work is equally distributed among them),

(c) average wait time for service.

3. Consider a B&D process with the follwoing (per hour) rates

$$\lambda_n = \frac{3.7}{1+n^2}$$

$$\mu_n = \begin{cases} \dfrac{12.4}{1+n} & \text{when} \quad n \geq 1 \\ 0 & \text{when} \quad n = 0 \end{cases}$$

and the initial value of 10. Confirm the existence of a stationary distribution, and find the long-run frequency of visits to State 0 (per 24-hour day) and their average duration (in minutes and seconds).

4. Without Maple, find the solution to

$$\sin(z) \cdot \dot{P}(z,t) = \mu \cdot \cos(z) \cdot P'(z,t)$$

(where μ is a constant) which meets

$$P(z,0) = \frac{\cos z}{\sin^2 z}$$

5. Without Maple, find the solution to

$$z \cdot \dot{P}(z,t) = \mu \cdot P'(z,t) + z \cdot P(z,t)$$

(where μ is a non-zero constant) which meets

$$P(z,0) = 1$$