MATH 4P85 SECOND MIDTERM NOVEMBER 23, 2017

Open book exam. **Duration: 1 hour**
Full credit given for correctly answering 6 (out of 10) questions.

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1. Consider a LGWI process with the following rates

$$\begin{aligned} \lambda_n &= 23 + 6.1n \text{ per hour} \\ \mu_n &= 6.1n \text{ per hour} \end{aligned}$$

starting with 11 members at 8:17. Compute

   (a) the expected value and standard deviation of the number of (surviving) immigrants and their descendents at 9:03,

   (b) the probability that the native sub-population (the initial members and their descendents) will survive (having at least one member) till 22:03.

2. Consider an M/M/1 queue with the average arrival rate of 12.3 customers per hour and the average service time of 15 minutes and 37 seconds. An arriving customer will join the queue with the probability of 0.75^n, where n is the number of customers ahead of him (including the one in service, if any). Find the long-run average

   (a) number of customers in the system,

   (b) time spent waiting for service.

3. Without Maple, find the solution to the following PDEs and the corresponding initial conditions. Provide details of your computation, including the form of the $g(x)$ function.

   (a)

$$\begin{aligned} z \cdot \dot{P}(z,t) &= P'(z,t) \\ P(z,0) &= z \end{aligned}$$

(b)

$$\begin{aligned} \dot{P}(z,t) &= P'(z,t) - 2P(z,t) \\ P(z,0) &= e^{-2z} \end{aligned}$$

4. Consider an M/M/10 queue with the average arrival rate of 12.3 customers per hour, the average service time of 47 minutes and 32 seconds, and the initial value of 11 customers (10 in service, 1 waiting). What is

   (a) the probability of at least three of the initial customers finishing their service *before* the next arrival,

   (b) the long-run frequency of visits to State 10 (all servers busy but no custmer waiting) per 24 hour day (assume non-stop operation).

5. Consider a B&D process with the following rates

$$\begin{aligned} \lambda_n &= \frac{1.03n}{n+1} \quad \text{per hour} \\ \mu_n &= \frac{n^2}{n^2+1} \quad \text{per hour} \end{aligned}$$

   starting in State 75. Compute the probability that the process

   (a) will get extinct,

   (b) won't ever reach State 60.