MATH 4P85 SECOND MIDTERM NOV. 16, 2018

Open book exam. **Duration: 1 hour**

Full credit given for 6 (out of 10) complete and correct answers

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1. Consider a LGWI process with the following rates

$$\lambda_n = 5.4\ n + 13.7 \ \text{ per hour}$$
$$\mu_n = 5.4\ n \ \text{ per hour}$$

and having 15 members at 8:37. Compute

(a) the expected time (use the xx:yy:zz format) of death of the last of the 15 *initial* members (ignore descendents and immigrants), and the corresponding standard deviation (in minutes and seconds),

(b) the expected number of immigrant *arrivals* (ignore their subsequent fate) before the last *initial member* dies (hint: recall Poisson process of random duration),

(c) the probability that the native sub-population becomes extinct between 14:12 and 23:09.

2. Consider the $M/M/6$ queue with the average arrival rate of 90.2 customers per hour, and the average service time of 3 minutes and 11 seconds. Currently, there are 2 customers waiting (all 6 servers are busy). Compute

(a) the expected time it will take until (for the first time from now) there is nobody waiting,

(b) the long-run proportion of time with all 6 servers busy,

(c) the long-run average time spent by a customer in the *system* (waiting *and* being served).

3. Consider a B&D process with the following (per *minute*) rates

$$\lambda_n = \frac{24.7}{1+n^2}$$

$$\mu_n = \begin{cases} \dfrac{3.4}{1+n} & \text{when} \quad n \geq 1 \\ 0 & \text{when} \quad n = 0 \end{cases}$$

and the initial value of 15. Compute

(a) the probability that, three transitions later, the process is in State 16,

(b) the long-run frequency of visits (per *hour*) to State 10.

4. Without Maple (assuming z is small and positive), find

(a) a general solution to

$$z \cdot \dot{P}(z,t) \cdot \tan z = z \cdot P'(z,t) + P(z,t)$$

(b) the solution which meets

$$P(z,0) = \frac{\sin^2 z}{z}$$

You must spell out what exactly did you use for the $g(x)$ function of Part a).