MATH 4P85 SECOND MIDTERM MARCH 27, 2005
Full credit given for three correct and complete answers.
Please, give all answers to four significant digit. Duration: 50 minutes

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1. Consider the following experiment: A random number (let us call it N) is drawn from a Poisson distribution with the mean of 7.46. This is then followed by flipping a coin till N heads are obtained (we don't flip when $N = 0$). Find:

   (a) The expected value and the corresponding standard deviation of the number of flips.

   (b) The probability that more than 20 flips will be needed.
   Hint: Use a composition of Poisson and Geometric distributions.

2. Find the general solution to

$$z\dot{P}(z,t) = P'(z,t) - P(z,t)$$

   Also, find the specific solution which meets

$$P(z,0) = \exp\left[\frac{(z+1)^2}{2}\right]$$

3. Consider a birth-and-death process with rates given by

$$\begin{aligned} \lambda_n &= 3.6\ n \qquad \text{per hour} \\ \mu_n &= 3.5\ n \qquad \text{per hour} \end{aligned}$$

   which is observed to be in State 13 at 8:07 am. Find:

   (a) The expected value and standard deviation of the number of members of this process at 9:24 am.

   (b) The probability that, at 9:24 am., the process has between 1 and 25 members (inclusive).

   (c) The probability that the process gets extinct between 9:15 and 11:30 am.

4. Consider an $M/M/6$ queue with the average service time of 17 min. and 42 sec., and customers arriving at an average rate of 19.3 per hour. Compute the long-run

   (a) server utilization factor,

   (b) average size of the line up,

   (c) average time a customer spends waiting for service,

   (d) percentage of time with all servers busy.

   (e) How many times a week (on the average) , does the process enter State 0 (all servers idle)? Assume a round-the-clock operation.

5. Consider an M/M/2 queue with 20 customers arriving on the average every hour, but walking away with the probability of $1-\frac{1}{2^m}$, where m is the number of people *waiting*. The average service time is 10 minutes.

   (a) Make a table of λ_n and μ_n rates for n from 0 to 6.
   Also, compute the long-run average number of

   (b) busy servers,

   (c) customers waiting for service.