MATH 4P85 FIRST MIDTERM FEBRUARY 8, 2005

Full credit given for three correct and complete answers.

Please, give all answers to four significant digit.

Open-book exam. Duration: 50 minutes

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1. Customers arrive at a gas station at the rate of 17.4 per hour. Of these, 68% are cars and 32% are trucks (including vans). Find the probability that

   (a) During the next half hour there will be more that 5 cars and (at the same time - this is a single question) fewer than 3 trucks arriving.

   (b) The tenth car of the day will arrive before the fifth truck.

   (c) The tenth car of the day will arrive more than half an hour after opening.

2. Consider an $M/G/\infty$ process with the arrival rate of 27 customers per hour and the probability density function of individual service times given by

$$g(x) = 3000x^2 \exp(-1000x^3)$$

   where $x > 0$ (in hours). Find

   (a) the long run (i.e. $t \to \infty$) average number of busy servers,

   (b) the long run proportion of time with more than 5 customers being serviced,

   (c) the probability that, fifteen minutes after opening (with no customers waiting), more than 5 customers have already completed their service.

3. Consider a cluster Poisson process with $\lambda = 6.8$ per hour, and the probability that a cluster contains n individuals is given by

$$\frac{1.2^{n-1}e^{-1.2}}{(n-1)!} \qquad n = 1,\ 2,\ 3,\ ...$$

   (note that this is a distribution of $1+X$, where X has the Poisson distribution with $\Lambda = 1.2$; it should be easy to construct the corresponding probability generating function).

(a) What is the expected valued and the standard deviation of the number of *clusters* arriving during the first half hour?

(b) What is the expected valued and the standard deviation of the number of *individuals* arriving during the first half hour?

(c) What is the probability that the number of individuals arriving during the first half hour will exceed 15?

4. Consider a Poisson process with $\lambda = 4.7$ arrivals per hour, observed for a length of time whose duration is random, and has the probability density function of

$$\frac{x^{10}}{10!}\exp(-x)$$

where $x > 0$ (in hours).

(a) Identify the last distribution (i.e. spell out its name, and specify the value of its parameters).

(b) Find the expected number of arrivals (during this time interval) and the corresponding standard deviation.

(c) What is the probability of getting more than 50 arrivals?

5. Suppose a store's customers arrive at a rate (per hour) which varies with time as follows

$$\lambda(t) = 14.2 + \frac{100t}{40 + t^3}$$

where t is the time (in hours), set to 0 at 8:00 o'clock (when the store opens). This means that noon corresponds to $t = 4$, etc. Find the probability that

(a) fewer than 8 customers arrive between 9:00 and 9:20,

(b) exactly 3 customers arrive before 8:10,

(c) the third customer of the day arrives before 8:10.