MATH 4P85 SECOND MIDTERM MARCH 22, 2005
Full credit given for three correct and complete answers.
Please, give all answers to four significant digit.
Open-book exam. Duration: 50 minutes

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1. Consider a birth-and-death process with rates given by

$$\begin{aligned} \lambda_n &= 3.5\, n \qquad \text{per hour} \\ \mu_n &= 3.6\, n \qquad \text{per hour} \end{aligned}$$

and the initial state equal to 13. Find:

   (a) The expected value and standard deviation of the number of members of this process 12 minutes later.

   (b) The probability that 12 minutes later there are fewer than 9 members left.

   (c) The expected time till extinction.

2. Consider a system consisting of 9 welders, working independently of each other. When any of them uses electricity, it is for a random time which is exponentially distributed with the mean of 18 seconds. Similarly, the intervals in between electricity usage are (for each of them) exponentially distributed with the mean of 27 seconds. Assume that the initial value of the process (the number of welders using electricity now) is 3. Compute:

   (a) The expected value and standard deviation of the number of welders using electricity 7 seconds later.

   (b) The probability that, 7 seconds later, there is more than 5 welders using electricity.

   (c) In the long run, what is the percentage of time with no electricity use, and how often does this happen (on the average, per hour)?

3. Consider an M/M/4 queue, with customers arriving at the rate of 2.3 per hour, and the average length of a service time being 1 hour and 12 minutes. Assuming the process has been running for a long time, compute:

(a) The average number of customers in the system (i.e. both in service and waiting).

(b) Percentage of time with all servers idle, and how often does this happen (on the average, per day)?

(c) The server utilization factor.

(d) Percentage of time with at least 3 servers busy.

4. Consider an M/M/1 queue with 15 customers arriving on the average every hour, and joining the line-up with the probability of $\frac{1}{\max(n,1)}$, where n is the number of customers in the system (with the probability of $1-\frac{1}{\max(n,1)}$, they walk away). The average service time is 12 minutes. Compute:

   (a) The server utilization factor.

   (b) The average number of people in the system, and the corresponding standard deviation.

5. Assume that a birth-and-death process has the following (per hour) rates

State:	0	1	2	3	4	5
λ_n	3.5	4.3	4.0	3.7	3.1	0
μ_n	0	2.9	3.1	3.6	4.2	4.9

   Assume that the process starts in State 5.

   (a) What is the expected value of the process 16 hours later (note that this is more than sufficient to reach the process' stationary mode), and the corresponding standard deviation?

   (b) In the long run, what percentage of time is spent in State 3, and how often is this state visited (on the average, per hour)?