MATH 4P85 SECOND MIDTERM MARCH 29, 2004
Full credit given for three correct and complete answers.
Please, give all answers to four significant digit.
Open-book exam. Duration: 50 minutes

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1. Consider a pure-death process with rates given by

$$\mu_n = 3.6\ n \qquad \text{per hour}$$

and the initial state equal to 13. Find:

   (a) The expected value and standard deviation of the number of members of this process 12 minutes later.

   (b) The probability that 12 minutes later there are fewer than 9 members left.

   (c) The expected time till extinction, and the corresponding standard deviation.

2. Consider a system with infinitely many servers, customers arriving at the rate of 17.4 per hour, and the service times having the exponential distribution with the mean of 12 minutes and 41 seconds. Assume that the initial value of the process (the number of customers waiting, when the service opens) is 24. Compute:

   (a) The expected value and standard deviation of the number of customers in the system 7 minutes after opening.

   (b) The probability that, 7 minutes after opening, there is more than 20 customers in the system.

   (c) In the long run, what is the percentage of time with all servers idling?

3. Consider an M/M/3 queue, with customers arriving at the rate of 2.3 per hour, and the average length of a service time being 1 hour and 12 minutes. Assuming the process has been running for a long time, compute:

   (a) The average size of the line-up.

(b) Percentage of time with no line-up.

(c) The server utilization factor.

(d) Percentage of time with all servers busy.

4. Consider an M/M/1 queue with 15 customers arriving on the average every hour, but walking away with the probability of $\left(\frac{n}{n+1}\right)^2$, where n is the number of people in the system. The average service time is 12 minutes. Compute:

   (a) The server utilization factor.

   (b) The average number of people in the system, and the corresponding standard deviation.

5. Assume that a Birth-and-Death process has the following (per hour) rates

State:	0	1	2	3	4	5
λ_n	0	4.3	4.0	3.7	3.1	0
μ_n	0	2.9	3.1	3.6	4.2	4.9

   (note that State 0 is absorbing). If the process starts in State 5, what is the expected time till absorption?