MATH 4P85   FIRST MIDTERM   FEBRUARY 10, 2014

One sheet of notes and a Maple workspace (loaded from a memory stick) containing any information are allowed.

Full credit given for 9 (out of 15) correct and complete answers.

Each answer (the final number, if nothing else) must be entered in your exam booklet.

Give all answers to at least four significant digit.   **Duration: 50 min.**

1. Customers arrive at a rate given by the following expression: $\lambda(t) = \dfrac{12.3}{1+t}$.
   Find

   (a) the probability of more than 6 arrivals between $t = 2$ and $t = 4$,

   (b) the covariance between the number of arrivals in the $(0, 2)$ time interval, and in the $(0, 4)$ time interval,

   (c) the expected time of the 3rd arrival (assuming the store opens at $t = 0$), and the corresponding standard deviation.

2. Suppose that customers arrive at the rate of 9.3 'clusters' per hour, where the size of each cluster has the regular (counting trials) geometric distribution with $p = \frac{1}{3}$. Find

   (a) the expected number of *customers* who arrive during the next 37 minutes, and the corresponding standard deviation,

   (b) the probability that the number of *customers* who arrive during the next 37 minutes is between 11 and 23 (inclusive),

   (c) assuming ('given') that exactly 7 *clusters* will arrive during the next 37 minutes , what is the conditional probability that at least one of them will have more than 4 customers.

3. Consider a $M/G/\infty$ queue with customers arriving at a rate of 13.2 per hour, and the service time of each customer having a uniform distribution between 2 and 7 minutes. Find

   (a) the probability that, 5 minutes after opening (with no customers waiting at the door), there are exactly 2 customers being served,

   (b) the long-run average number of busy servers,

   (c) the probability that the service times of the first 3 customers are all shorter than 5 minutes (each).

4. Customers arrive at a dollar store (every item costs $1) at a constant rate of 19.6 per hour, but each of them will make a purchase with the probability of 71% (otherwise, he/she will only browse). A buying customer purchases between 1 and 7 items, according the following probabilities:

# of items:	1	2	3	4	5	6	7
Pr:	0.05	0.10	0.20	0.30	0.20	0.10	0.05

Compute the probability that

(a) during the next 45 minutes, the store gets at least 10 buying customers and (at the same time - this is a single question) no more than 7 browsers,

(b) by the time the store gets its 7th 'buyer', it has had no more than 3 browsers.

(c) Also: Find the expected value of the total amount spent by customers who arrive during the next 45 minutes, and the corresponding standard deviation.

5. Consider a 2-dimensional Poisson process with $\lambda = 17.2$ 'dots' per meter^2
Find the *expected value* and *standard deviation* of

(a) the number of 'dots' inside the triangle with vertices $(-2, 3)$, $(3, -2)$ and $(1, 1)$,

(b) the distance from the origin to its second nearest 'dot'.

(c) Also: Given that there are exactly 50 dots found inside the unit circle centered on the origin, what is the conditional probability that more than 27 of them will be to the left of the y axis (i.e. having a negative x coordinate).