1.

(a) Use LGWI formulas with $i = 0$:

$$\begin{aligned}\frac{a\ q_t}{\mu\ p_t} &= \frac{23}{6.1} \cdot 6.1 \cdot \frac{46}{60} = 17.633 \\ \sqrt{\frac{a\ q_t}{\mu\ p_t^2}} &= \sqrt{17.633 \cdot \left(1 + 6.1 \cdot \frac{46}{60}\right)} = 10.005\end{aligned}$$

(b) Use Linear Growth formula:

$$1 - r_t^i = 1 - \left(\frac{6.1 \cdot \left(13 + \frac{46}{60}\right)}{1 + 6.1 \cdot \left(13 + \frac{46}{60}\right)}\right)^{11} = 12.209\%$$

2.

(a)

$$\frac{\sum_{n=0}^{\infty} n \prod_{j=1}^{n} \frac{\lambda_{j-1}}{\mu_j}}{\sum_{n=0}^{\infty} \prod_{j=1}^{n} \frac{\lambda_{j-1}}{\mu_j}} = \frac{\sum_{n=0}^{20} n \prod_{j=1}^{n} 12.3 \cdot 0.75^{j-1} \cdot \left(\frac{15}{60} + \frac{37}{3600}\right)}{\sum_{n=0}^{20} \prod_{j=1}^{n} 12.3 \cdot 0.75^{j-1} \cdot \left(\frac{15}{60} + \frac{37}{3600}\right)} = 4.5625$$

Verify that replacing ∞ by 20 is good enough!

(b) Find $\mathbb{E}(U)$ first (divide the previous answer by λ_{av} - Little's formula), then subtract $\frac{1}{\mu}$.

$$\lambda_{\text{av}} = 12.3 \cdot \frac{\sum_{n=0}^{20} 0.75^n \prod_{j=1}^{n} 12.3 \cdot 0.75^{j-1} \cdot \left(\frac{15}{60} + \frac{37}{3600}\right)}{\sum_{n=0}^{20} \prod_{j=1}^{n} 12.3 \cdot 0.75^{j-1} \cdot \left(\frac{15}{60} + \frac{37}{3600}\right)} = 3.7998$$

implying

$$\frac{4.5625}{3.7998} - \left(\frac{15}{60} + \frac{37}{3600}\right) = 0.94044 \text{ hours}$$

3.

(a)

$$g\left(t + \int z\ dz\right) = g\left(t + \frac{z^2}{2}\right)$$

implying

$$g(x) = \sqrt{2x}$$

Thus

$$P(z,t) = \sqrt{2t + z^2}$$

(b)

$$g\left(t+\int dz\right)\cdot\exp\left(2\int dz\right)=g(t+z)\cdot e^{2z}$$

implying

$$g(x)=e^{-4x}$$

Thus

$$P(z,t)=e^{-4(t+z)}\cdot e^{2z}=e^{-4t-2z}$$

4.

$$\mu=\frac{1}{\frac{47}{60}+\frac{32}{3600}}=1.2623$$

(a)

$$\frac{\mu_{11}}{\lambda_{11}+\mu_{11}}\cdot\frac{\mu_{10}}{\lambda_{10}+\mu_{10}}\cdot\frac{\mu_9}{\lambda_9+\mu_9}=\left(\frac{10\cdot\mu}{12.3+10\cdot\mu}\right)^2\cdot\frac{9\cdot\mu}{12.3+9\cdot\mu}=12.317\%$$

(b)

$$\begin{aligned}\rho &= \frac{12.3}{\mu}=9.744\bar{3} \qquad \text{(implying the process is stable)}\\ p_{10} &= \frac{\rho^{10}}{10!\left(\sum_{k=0}^{9}\frac{\rho^k}{k!}+\frac{\rho^{10}}{10!\left(1-\frac{\rho}{10}\right)}\right)}=0.023231\\ & \quad p_{10}\cdot(12.3+10\mu)\cdot 24=13.896 \ \text{ per day}\end{aligned}$$

5.

(a)

$$1-\frac{\sum_{n=0}^{74}\prod_{k=1}^{n}\frac{k\cdot(k+1)}{(k^2+1)\cdot 1.03}}{\sum_{n=0}^{900}\prod_{k=1}^{n}\frac{k\cdot(k+1)}{(k^2+1)\cdot 1.03}}=33.891\%$$

Make sure 900 is large enough.

(b)

$$\frac{\sum_{n=60}^{74}\prod_{k=1}^{n}\frac{k\cdot(k+1)}{(k^2+1)\cdot 1.03}}{\sum_{n=60}^{900}\prod_{k=1}^{n}\frac{k\cdot(k+1)}{(k^2+1)\cdot 1.03}}=25.717\%$$