MATH 4P85 SECOND MIDTERM NOVEMBER 16, 2016
Open-book exam. Use of Maple and calculator permitted.
Full credit given for 6 (out of 10) correct answers. **Duration: 1 hour**

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1. Consider a LGWI process with the following rates

$$\begin{aligned} \lambda_n &= 8.12n + 11.43 \text{ per year} \\ \mu_n &= 9.04n \text{ per year} \end{aligned}$$

which starts with 17 initial members. Assuming that a year has 365 days compute

(a) the expected value of the process two years later and the corresponding standard deviation,

(b) the expected time of death of the last initial member, and the corresponding standard deviation (give both answers in days and hours, e.g. 203 days and 13 hours),

(c) the expected time till extinction of the native sub-population (initial members and their descendents), and the corresponding standard deviation (give in years and days, e.g. 2 years and 107 days),

(d) the long-run proportion of time with at least 4 living immigrants (**not** counting their descendents),

(e) the long-run proportion of time with more than 12 living immigrant sub-population (i.e. counting the descendants as well).

2. Consider 15 welders who, individually and independently of each other, alternate using and not using current. The corresponding intervals are also independent of each other, and are exponentially distributed with the mean of 18 seconds (drawing current), and 1 minute and 7 seconds (idling). At 9:42:37, exactly six of the welders (we say that the process is in State 6) are using current; a transition is defined as a change of state. Compute

(a) the probability that, 3 transitions later, the process is in State 5,

(b) the expected value of the process at 9:43:12 and the corresponding standard deviation,

(c) the long-run frequency of visits to State 0 (per hour), and their average duration (in seconds).

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3. Consider the following PDE

$$\frac{\partial P(z,t)}{\partial t}\sin^2 z + \frac{1}{2}\frac{\partial P(z,t)}{\partial z}\sin(2z) + P(z,t)\cos^2 z = 0$$

(a) Without using dsolve, and providing sufficient details, find its general solution,

(b) and the specific solution which meets

$$P(z,0) = \sin z$$

making it clear what your $g(x)$ function is.