1. Find the best (least-square) values for the four parameters of the following model

$$y_i = a \sin(b \cdot x_i + c) + d + \varepsilon_i$$

where ε_i are independent from $\mathcal{N}(0, \sigma)$, by fitting it to the supplied data. Also compute the standard error of each of your estimates, and the corresponding (approximate) correlation matrix. Finally, estimate the value of σ, and give the corresponding standard error.

2. Assuming that the given sequence of observations has been generated from the usual Yule model

$$X_i = \alpha_1 X_{i-1} + \alpha_2 X_{i-2} + \varepsilon_i$$

where ε_i are independent, normally distributed with zero mean and standard deviation σ, find the ML estimators of α_1, α_2 and σ (note that there is no μ).

3. Using Bartlett window with $M = 30$, estimate the power spectrum of the given sequence of X_i values. Using this result, predict (and plot) the power spectrum of a new sequence, created by

$$Y_i = X_i + 0.75X_{i-1} + 0.5X_{i-2} + 0.25X_{i-3}$$

(make sure that the average value of your spectral density is 1). Verify your prediction by finding (and plotting) the empirical power spectrum of the new sequence (using the same Bartlett window). Each power-spectrum function should be expanded in terms of $\cos\beta$, $\cos 2\beta$, $\cos 3\beta, \ldots$

4. Using the set of data provided, find the ML estimators for μ, σ and ρ of the following model:

$$X_i = \mu + \rho(X_{i-1} - \mu) + \varepsilon_i$$

where ε_i are IID from $\mathcal{N}(0, \sigma)$, and $X_0 \equiv X_{50}$.

5. Consider the following ARMA model:

$$X_i = 0.93X_{i-1} + \varepsilon_i - \varepsilon_{i-1} + \varepsilon_{i-2} - \varepsilon_{i-3}$$

where ε_i are independent, Normally distributed with the mean equal to zero and $\sigma = 1.7$.

   (a) Compute and plot ρ_k for $k = 1, 2, \ldots 50$.

   (b) Find (express it as a rational function of $\cos\beta$, $\cos 2\beta$ and $\cos 3\beta$) and plot the process' spectral density.

   (c) Find

$$\Pr(X_{313} < -4.9 \mid X_{311} = -3.5 \cap X_{310} = 2.4 \cap X_{309} = -1.8)$$