1. Consider paying \$5 to play the following game: 6 cards are dealt randomly from a standard deck of 52 cards, and you get paid \$2 for each spade and \$3 for each ace (\$5 for the ace of spades). Find

   (a) the expected net win (loss) and the corresponding standard deviation (use formulas, not PGF),

   (b) and the probability of winning at least \$1 (hint: now, construct the corresponding PGF first).

2. Given the following joint PDF of a bi-variate Normal distribution

$$\frac{1}{\pi} \cdot \exp\left(-\frac{16x^2 + 9y^2 - 20xy + 6x - 4y + c}{288}\right)$$

   where c is the appropriate constant, find the

   (a) marginal distribution of Y (we know it's Normal, just spell out its two parameters),

   (b) conditional distribution of Y given $X = \mathbf{x}$ (we know it's Normal, just spell out its two parameters),

   (c) probability that $X > Y$.

3. Given the following *joint* MGF of a bi-variate distribution of X and Y

$$2 \cdot \frac{u\ e^t - t\ e^u + t - u}{t\ u\ (t - u)}$$

   find the $2^{nd}, 3^{rd}$ *central* moment of X and Y, and also the skewness of Y.

4. Find the characteristic function of a distribution with the following PDF

$$f(x) = \begin{cases} 2(1-x) & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

   Convert it to the characteristic function of $\bar{X}$, i.e. of the sample mean of 20 independent values from this distribution. Using Maple, find the corresponding PDF of $\bar{X}$ (don't quote it) and plot the difference between this (exact) PDF and the PDF of the corresponding Normal approximation.

5. Find and *identify* the PDF of $X + Y$, where X and Y are independent random variables of the following type:

   (a) X (Y) is Normally distributed, with the mean of 3 (-5) and the standard deviation of 4 (3) respectively,

   (b) X (Y) has a Cauchy distribution with the median of 3 (-5) and quartile deviation of 4 (3) respectively,

   (c) both X and Y are drawn from an Exponential distribution with the mean of 2.5.