1. Express

$$\sum_{i=1}^{n} \left(x_i^2 + y_i^2 - 2\rho x_i y_i \right)$$

in terms of

$$\begin{aligned}
\bar{x} &= \frac{\sum_{i=1}^{n} x_i}{n} \\
\bar{y} &= \frac{\sum_{i=1}^{n} y_i}{n} \\
s_1 &= \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}} \\
s_2 &= \sqrt{\frac{\sum_{i=1}^{n} (y_i - \bar{y})^2}{n}} \\
r &= \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{n \cdot s_1 s_2}
\end{aligned}$$

2. Convert

$$\int_0^\infty \int_0^y x^{n-2} y^{n-2} \exp\left(-\frac{n(x^2 + y^2 - 2\rho r x y)}{2(1-\rho^2)} \right) dx\ dy$$

where n is an integer and ρ and r are two constants, each having a value between -1 and 1, to a double integral in terms of new variables $u = xy$ and

$$v = \frac{1}{2} \left(\frac{x}{y} + \frac{y}{x} \right)$$

You may assume (correctly, as it turns out) that there is a one-to-one correspondence between the old and the new pair of variables. Finding the *inverse* of the Jacobian may be a helpful 'shortcut'.

3. Convert

$$\int_1^\infty \frac{(v - \rho r)^{n-1}}{\sqrt{v^2 - 1}} dv$$

into an dz integral by the following substitution

$$v = \frac{1 - \rho r z}{1 - z}$$

4. Using the formula from class, find

$$\frac{\partial \det \mathbb{A}}{\partial a_{23}}$$

where $\mathbb{A}$ is a general 4 by 4 matrix. Verify your answer at

$$\mathbb{A} = \begin{bmatrix} 1 & -3 & 5 & 2 \\ 7 & 4 & -2 & 1 \\ 6 & -2 & 3 & 5 \\ -3 & 0 & 4 & 6 \end{bmatrix}$$