1. Sampling from a bivariate Normal distribution, the distribution of the sample correlation coefficient r is *approximately* Normal, with the mean of ρ and the standard deviation of $\frac{1-\rho^2}{\sqrt{n}}$.

   (a) Assuming that $\rho = 0.91$ and $n = 210$, plot the the difference between the exact and approximate PDFs; find the largest possible error of this approximation.

   (b) Similarly, one can show that an approximate PDF of

$$Z \equiv \frac{\mathsf{arctanh}\ r - \text{arctanh}\ \rho - \frac{\rho}{2n}}{\sqrt{\frac{1}{n} + \frac{6-\rho^2}{2n^2}}}$$

   is

$$\frac{\exp\left(-\frac{z^2}{2}\right)}{\sqrt{2\pi}} \cdot \left(1 + \frac{z^4 - 6z^2 + 3}{12n}\right)$$

   (note that this can be easily transformed to yield an approximate PDF of r).

   Using the same parameters as in a), plot the difference between the exact and the (Part b) approximate PDF of r; find the largest possible error of this approximation.

   In case you don't relate to hyperbolic functions

$$\text{arctanh}\ x \equiv \frac{1}{2} \ln \frac{1+x}{1-x}$$

2. Consider a RIS of size n from the $\mathcal{N}(\mu, \sigma)$ distribution. Based on the joint PDF of the n observations (we will denote them X_1, X_2, ...X_n), find (using the geometrical method) the bi-variate PDF of $U = \bar{X}$ and

$$V \equiv \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{\sigma^2}$$

   Are U and V independent? Identify the resulting two marginals (verify that each has the correct normalizing constant).