1. Assuming a three-parameter Markov model with the values of the parameters known, the long-run proportion of time that the corresponding 'smart' prediction of the next value will come closer to the actual observation of that value than the prediction which uses μ only (ignoring the current value) is given by

$$\Pr\left(|U+V| \ > |V|\right)$$

where $U \epsilon \mathcal{N}\left(0, \frac{|\rho|}{\sqrt{1-\rho^2}}\right)$ and $V \epsilon \mathcal{N}(0,1)$ are independent. Find a formula for this probability and plot it as a function of ρ (note that the answer does not depend on μ, σ). What is this proportion when $\rho = -0.6$?

2. For a three-parameter Markov model, find the set of normal equations for the ML estimators of these parameters (based on a sample of N consecutive and equilibrated observations denoted $x_1, x_2, \ldots x_N$). Solve these equations (numerically and, if necessary, iteratively) using the provided data.

3. Consider the Yule model with $\alpha_1 = 1.32$, $\alpha_2 = -0.53$, $\sigma = 0.74$ and n large (the process has equilibrated). Compute:

   (a) $\Pr(X_n < 1.1)$,
   (b) $\Pr(X_n < 1.1 \mid X_{n-1} = 0.4)$,
   (c) $\Pr(X_n < 1.1 \mid X_{n-1} = 0.4 \cap X_{n-2} = -1.3)$,
   (d) $\Pr(X_n < 1.1 \mid X_{n-1} = 0.4 \cap X_{n-2} = -1.3 \cap X_{n-3} = 0.9)$.

   Note that you can use 'shortcuts' to answer c and d.

4. For the Yule model, find (and simplify) the partial correlation coefficient between X_{n+1} and X_{n-1} given the value of X_n.