1. For a four-parameter (α_1, α_2, μ and σ) Yule model, find and then solve the set of normal equations for the MLEs of these parameters, based on a *given* set of N consecutive observations.

2. Consider the following autoregressive model:

$$X_n = \frac{23}{30} X_{n-1} + \frac{52}{75} X_{n-2} - \frac{8}{15} X_{n-3} + \varepsilon_n$$

where $\varepsilon_n \in \mathcal{N}(0, \sigma = 4.2)$.

   (a) Find an exact (no decimals) formula for ρ_k. What is the smallest k your formula is good for?

   Compute:

   (b) $\Pr(X_{116} < 7.9 \mid X_{115} = 6.1 \cap X_{113} = -2.8)$,

   (c) $\Pr(X_{117} < 7.9 \mid X_{115} = 6.1 \cap X_{113} = -2.8 \cap X_{112} = 4.5)$,

   (d) $\mathbb{E}\left(X_{113} X_{115} X_{116}^2\right)$. Hint: Construct the joint MGF first.

3. Consider the following ARMA model:

$$X_i - \frac{5}{3} X_{i-1} + \frac{17}{18} X_{i-2} = \varepsilon_i + 2\varepsilon_{i-1} - 3\varepsilon_{i-2}$$

where ε_i are independent, Normally distributed with mean of zero and $\sigma = 3.1$.

   (a) Find an exact formula for ρ_k. What is the smallest k your formula is good for?

   Compute:

   (b)

$$\Pr(X_{312} > -5.3 \mid X_{311} = -4.1 \cap X_{310} = -2.8)$$

   (c)

$$\Pr(X_{312} > -5.3 \cap X_{313} < 1.7 \mid X_{311} = -4.1 \cap X_{310} = -2.8)$$

   (d) and the partial correlation coefficient between X_{312} and X_{308} given that $X_{311} = -4.1$, $X_{310} = -2.8$ and $X_{309} = 3.6$.