Note that 'find' means you should *spell out* the corresponding number, formula, sequence, model etc.

1. Find and plot the spectral density of the folowing ARMA model

   $$X_n = 0.7X_{n-1} - 0.8X_{n-2} + 0.6X_{n-3} + \varepsilon_n - 3.2\varepsilon_{n-1} + 2.4\varepsilon_{n-2} - 4.1\varepsilon_{n-3}$$

   with $\sigma = 1.4$. Also compute

   $$\Pr(X_{753} > X_{752} \mid X_{751} = -11.3 \cap X_{750} = 8.7 \cap X_{749} = -9.4)$$

2. Generate and plot 700 consecutive and *equilibrated* values using the ARMA model of Question 1. Then, using the sin/cos formula, find and plot the corresponding $\hat{\omega}(\beta)$ for $\beta = 0$ to π (inclusive), in steps of $\frac{\pi}{40}$. Finally, using the $\hat{\rho}_j$ formula, find and plot $\hat{\omega}(\beta)$ for all $\beta \in [0, \pi]$. Compare with the theoretical curve of Question 1 and comment on how well (or not) the empirical densities estimate the 'true' $\omega(\beta)$.

3. Consider the following spectral density

   $$\omega(\beta) = c \cdot \frac{3 - 2\cos\beta + \cos 2\beta - 2\cos 3\beta}{3 + \cos\beta + 2\cos 2\beta + 2\cos 3\beta}$$

   (a) Verify that it corresponds to a stationary ARMA model,

   (b) find all ARMA models (short of σ) having the above spectrum,

   (c) compute and plot ρ_1 to ρ_{20}.

4. Find all ARMA models (including σ) which yield

   $$\begin{aligned} \rho_k &= 0.75^k \cos 2k - k \cdot (-0.3)^k \sin 2k \\ V &= 6 \end{aligned}$$