

1. Use the following *spectral* window

$$\hat{\omega}_{\text{smooth}}(\beta) = \frac{\sum_{j=-3}^3 \hat{\omega}\left(\beta + \frac{j\pi}{40}\right)}{7}$$

to smooth the sin/cos version of $\hat{\omega}(\beta)$ obtained in Question 2 of the previous assignment. Print (the 41 values of $\hat{\omega}_{\text{smooth}}$) and plot the answer together with the theoretical spectral density (from Question 1).

Repeat by using the $\hat{\rho}$ version of $\hat{\omega}$ and the corresponding lag window (spell out its form) instead; this time plot the answer as a *continuous* function of β , but also verify getting the same results as the spectral window at the original 41 discrete values of β .

2. Smooth out the $\hat{\rho}$ (or ‘continuous’) version of $\hat{\omega}$ of same data using Tukey’s lag window with the ‘best’ possible choice of M (the one which results in the closest match to the theoretical $\omega(\beta)$, judged by simple eyeballing).
3. Repeat (plotting the resulting empirical spectral density as a continuous function of β , together with the theoretical spectrum) using the ‘uniform’ kernel

$$g(w) = \begin{cases} \frac{1}{2s} & \text{when } -s < w < s \\ 0 & \text{otherwise} \end{cases}$$

and similarly chosen ‘optimal’ s .

Find and plot the corresponding lag window, and verify that such lag windowing yields the same results.

4. Repeat the previous question using the following ‘normal’ kernel

$$g(w) = \frac{\exp\left(-\frac{w^2}{2s^2}\right)}{\sqrt{2\pi s^2}}$$