1. Let X_1, X_2,...X_5 have a multivarite Normal distribution with respective means of 3.5, -4.5, 0.5, 2.0, and 1.6, and the variance-covariance matrix of

$$\mathbb{V} = \begin{bmatrix} 32 & -4 & 10 & -8 & -11 \\ -4 & 37 & -1 & 1 & -12 \\ 10 & -1 & 7 & -8 & -2 \\ -8 & 1 & -8 & 15 & 10 \\ -11 & -12 & -2 & 10 & 32 \end{bmatrix}$$

(verify that it is positive definite).

   (a) What is the conditional distribution of X_1, X_2, given that $X_3 = -2.5$, $X_4 = 1.3$ and $X_5 = 4.0$.

   (b) Find a 5×5 matrix $\mathbb{B}$ such that $\mathbb{B}\mathbb{B}^T = \mathbb{V}$. Verify your answer.

2. Compute approximate ($\frac{1}{N}$ accurate) values of the standard errors of $\hat{V}$, r_1, r_2 and r_3, using the Yule model with $\alpha_1 = 1.4$, $\alpha_2 = -0.9$, $\sigma = 1$ and $N = 1000$.

3. For the circular Markov model with $\rho = -0.83$, $\sigma = 3.7$ and $X_{60} \equiv X_0$, compute and plot all 60 serial correlation coeficients. Also, compute $\Pr(X_1 > 0.87 \mid X_3 = -1.02 \cap X_{59} = 1.14)$.

4. Smooth out the empirical spectrum of the given set of data by dividing the 0 to π interval into 30 subintervals and using the following weighted averaging (spectral window):

$$\tilde{\omega}(\beta_i) = \frac{\hat{\omega}(\beta_{i-2}) + 2\hat{\omega}(\beta_{i-1}) + 3\hat{\omega}(\beta_i) + 2\hat{\omega}(\beta_{i+1}) + \hat{\omega}(\beta_{i+2})}{9}$$

(plot and print both the raw and smoothed out spectrum).

Find the lag window which achieves that same smoothing, plot and print the corresponding λ values, and verify that the two results are identical (print the resulting set of the 31 $\tilde{\omega}(\beta_i)$ values).

5. Consider the following spectral density of an ARMA model:

$$\omega(\beta) = c \cdot \frac{41 - 68\cos(\beta) + 38\cos(2\beta) - 13\cos(3\beta) + 2\cos(4\beta)}{367 + 276\cos(\beta) + 72\cos(2\beta) + 60\cos(3\beta) + 25\cos(4\beta)}$$

   (a) Find the value of c, and of the first 25 serial correlation coefficients.

   (b) What is the actual ARMA model (find *all* possible solutions).