

1. Find all ARMA models having the following serial correlation coefficients

$$\rho_k = 0.9^k - \frac{k}{3^k}$$

2. Find a $\frac{1}{N}$ -accurate formula for the bias of r_2 , assuming the Yule model (simplify the answer).

3. Using the given empirical spectrum consisting of pairs of values of β and the corresponding $\hat{\omega}(\beta)$, smooth it out using the following spectral window

$$\hat{\omega}_{sm}(\beta) = \frac{1}{13}\hat{\omega}\left(\beta - \frac{2\pi}{M}\right) + \frac{3}{13}\hat{\omega}\left(\beta - \frac{\pi}{M}\right) + \frac{5}{13}\hat{\omega}(\beta) + \frac{3}{13}\hat{\omega}\left(\beta + \frac{\pi}{M}\right) + \frac{1}{13}\hat{\omega}\left(\beta + \frac{2\pi}{M}\right)$$

where M is the number of subintervals (print and plot the new values). Find the corresponding lag window (in terms of λ_k).

4. Using the given set of consecutive observations, plot its ‘continuous’ empirical spectral density. Then, smooth out this density using the following kernel

$$G(z) = \begin{cases} \frac{75}{16} \cdot (25z^2 - 1)^2 & -\frac{1}{5} < z < \frac{1}{5} \\ 0 & \text{otherwise} \end{cases}$$

by finding the corresponding lag window first.

5. Using the same set of observations (Question 4), find ML estimates of the parameters α_1 , α_2 , α_3 and σ , assuming the *regular* AR(3) model (i.e. $\mu \equiv 0$).