MATH 5P84 FINAL EXAM 2012

1. For the following ARMA model

$$(X_n - 5) = 1.2(X_{n-1} - 5) - .8(X_{n-2} - 5) + .4(X_{n-3} - 5) + \varepsilon_n - \varepsilon_{n-1}$$

where ε_n are independent, normally distributed with the mean of 0 and the *variance* of 7.3, find the theoretical value of the common variance V and of the first 10 correlation coefficients. Also: what is the (*bivariate*) conditional distribution of X_{180} and X_{182} given $X_{176} = 6.3$, $X_{177} = 4.9$ and $X_{179} = 5.1$?

2. Find a basic ($\mu = 0$) ARMA model having the following spectrum

$$\omega(\beta) = c \cdot \frac{20 + 30\cos(2\beta) + 12\cos(4\beta) + 2\cos(6\beta)}{3.24 - 4.96\cos(\beta) + 2.56\cos(2\beta) - 0.8\cos(3\beta)}$$

and $V = 14.3$.

3. This is a continuation of the previous question: find a formula for the corresponding ρ_k (spell out the exceptional cases as well).

4. Find the lag window corresponding to the following kernel

$$g(u) = \begin{cases} \frac{15}{16s}\left(1 - 2(\frac{u}{s})^2 + (\frac{u}{s})^4\right) & -s < u < s \\ 0 & \text{otherwise} \end{cases}$$

(spell out the λ_k formula), and use it to smooth the empirical spectrum based on the given set of observations (choose $s = 0.4$; plot both the raw and smoothed spectrum in one graph - use the 'thickness=4' option for the latter and make it a full-page plot to be attached to your booklet).

5. For the *circular* Markov model with $\rho = -0.84$, $\sigma = 2.8$ and $n = 15$, find (print and plot) ρ_k for k from 0 to 15, the common variance V, and the (*bivariate*) conditional distribution of X_3 and X_4 given $X_1 = 3.2$ and $X_{14} = -2.3$.