1. Find an ARMA model with the following serial correlation coefficients

$$\rho_k = \cos(\frac{k}{3}) \cdot 0.95^k - k \cdot 0.2^k$$

According to this model (use $\sigma = 0.86$), what is the

$$\Pr(X_{223} > 3 \mid X_{222} = 1.13 \cap X_{220} = -0.84 \cap X_{219} = -2.77).$$

2. Given the following spectral density of an ARMA model (verify that is is 'legitimate')

$$\omega(\beta) = c \cdot \frac{1 - 0.15\cos(\beta) + 0.41\cos(2\beta) - 0.55\cos(3\beta)}{1 + 0.27\cos(\beta) - 0.18\cos(2\beta)}$$

where c is the appropriate constant, find the corresponding *general* formula for ρ_k. Assuming that V (the common variance of all X's) is 2.16, what is the *joint* distribution of X_{113}, X_{115} and X_{116} given that $X_{112} = -3.4$ and $X_{111} = 2.6$.

3. Smooth the given empirical spectrum (consisting of pairs of values of β and the corresponding $\hat{\omega}(\beta)$) using the following spectral window

$$\hat{\omega}_{sm}(\beta) = -\frac{1}{11}\hat{\omega}(\beta - \frac{2\pi}{M}) + \frac{3}{11}\hat{\omega}(\beta - \frac{\pi}{M}) + \frac{7}{11}\hat{\omega}(\beta) + \frac{3}{11}\hat{\omega}(\beta + \frac{\pi}{M}) - \frac{1}{11}\hat{\omega}(\beta + \frac{2\pi}{M})$$

where M is the number of subintervals (print the resulting $M + 1$ values). Also display $\hat{\omega}(\beta)$ before and after smoothing (in two separate graphs). Find the corresponding lag window (give the corresponding λ_k formula - no need to do the smoothing again).

4. Using the given set of consecutive observations, plot its 'continuous' empirical spectral density. Then, smooth out this density using the following kernel

$$G(z) = \begin{cases} 3 \cdot \big(1 + \cos(6\pi z)\big) & -\frac{1}{6} < z < \frac{1}{6} \\ 0 & \text{otherwise} \end{cases}$$

by using the appropriate lag window (spell out the λ_k formula), and plot (separately) the new density. Would you say that this kernel is too narrow or too wide? Reduce its width by a factor of $\frac{3}{4}$ and redo the exercise.

5. Using the given set of consecutive observations, find ML estimates of the parameters α_1, α_2, α_3, μ and σ, assuming the AR(3) model. Verify that the results represent a *stable* process.