1. Find ML estimates of the five parameters (α_1, α_2, α_3, μ and σ) of the AR(3) model, based on the posted (equilibrated) data. Hint: Use 'fsolve' to find the three α's and 'solve' to get μ and σ, and iterate.

   Using these estimates, find (and plot) the corresponding 'theoretical' spectral density (verify that your solution is stationary) and the value of the common variance V.

2. Adjust the data from Q1 by subtracting, from each X, the corresponding sample mean $\bar{X}$. Then, plot the resulting 'continuous' empirical spectral density of the new data, smoothed out

   (a) by using the following kernel

   $$g(u) = \begin{cases} 100u + 10 & -\frac{1}{10} \le u \le 0 \\ 10 - 100u & 0 \le u \le \frac{1}{10} \\ 0 & \text{otherwise} \end{cases}$$

   (spell out the formula for the corresponding λ_k - even if you choose not to use it - and plot its first 200 values),

   (b) and also by using the following lag window

   $$\lambda_k = \begin{cases} 1 & k \le 20 \\ 1 - \dfrac{k-20}{20} & 20 \le k \le 40 \\ 0 & \text{otherwise} \end{cases}$$

3. Assuming that the data provided for this question represent consecutive (and equilibrated) observations generated by

   $$X_n = -0.9X_{n-1} + \varepsilon_n - 0.9\varepsilon_{n-1}$$

   where the εs are IID from a Normal distribution with zero mean and variance equal to 10,

   (a) compute the corresponding r_1 and its (approximate) bias and standard error - compare with the true ρ_1.

   (b) Also evaluate the following estimator of V

   $$\hat{V} = \frac{\sum_{i=1}^{N} X_i^2}{N}$$

   and find its approximate standard error (here, you will have to derive your own formula) - compare with the actual value of V.

4. Assuming that Z_1, Z_2, Z_3, Z_4 and Z_5 is a random independent sample of size 5 from $\mathcal{N}(0, 1)$, and that

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{bmatrix} = \begin{bmatrix} 0 & 3 & -2 & 1 & 4 \\ -3 & 1 & 0 & 1 & 3 \\ -2 & 1 & 4 & 4 & -2 \\ 3 & 5 & -1 & -2 & 0 \\ 2 & -3 & 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{bmatrix} + \begin{bmatrix} 2 \\ -3 \\ 1 \\ 4 \\ -2 \end{bmatrix}$$

find the variance-covariance matrix of the five X's. Also compute

(a) $\Pr\left(X_3 > 2.4 \mid X_1 = -1.6 \cap X_2 = 3.8 \cap X_5 = 5.3\right),$

(b) $\Pr\left(|X_3 - X_2| < 1.3 \mid X_1 = -1.6 \cap X_5 = 5.3\right),$

(c) $\mathbb{E}\left((X_3 - 4)^2 \cdot X_4\right).$

5. Consider the following (theoretical) spectral-density function:

$$w(\beta) = c \cdot \frac{160000 \sin^4(\beta)}{361 + 32400 \cos^2(\beta)}$$

Find

(a) the corresponding ARMA model,

(b) the value of $\dfrac{\sigma^2}{V}$ where V is the common variance (after equilibration),

(c) a general (and exact) formula for ρ_k. If your formula does not cover all positive integers, make sure to also quote the exact values of ρ_1, ρ_2, ... (until the formula 'takes over'). Partial mark given for a list of ρ_1, $\rho_2, ...\rho_{10}$.