Each subquestion is worth 4 marks. No need for a formal report.

1. Let X_1, X_2, ... X_5 have a multivariate Normal distribution with the respective means of 3, -2, 0, 4 and -1, and the following variance-covariance matrix

$$\mathbb{V} = \begin{bmatrix} 24 & 9 & -15 & -13 & 0 \\ 9 & 12 & 1 & -4 & -10 \\ -15 & 1 & 24 & 11 & -14 \\ -13 & -4 & 11 & 14 & -5 \\ 0 & -10 & -14 & -5 & 18 \end{bmatrix}$$

   (a) Find
$$\Pr(X_3 > 2 \cap X_5 < -1 \mid X_2 = -0.9 \cap X_4 = 2.6)$$

   (b) Compute (using Digits:=30) the exact (in the sense of 'no approximation') probability that $r_{3,5} < -0.7$, where $r_{3,5}$ is the ML estimator of $\rho_{3,5}$ based on a RIS of size 67 from the above distribution.

   (c) Generate such a RIS of 67 quintuplets (no need to print them, but let Maple print your $\mathbb{B}$ matrix); based on this sample compute the value of $r_{3,5}$ and, using the Fisher transformation, construct the correspondingly 95% confidence interval for $\rho_{3,5}$.

2. Using the provided data (of 1000 equilibrated, consecutive values generated from an ARMA model)

   (a) plot (and, in Maple, print the corresponding values) of the corresponding discrete empirical spectrum (use 40 subintervals) smoothed out by the following spectral window: $W_j = \ < \frac{1}{16}, \frac{2}{16}, \frac{3}{16}, \frac{4}{16}, \frac{3}{16}, \frac{2}{16}, \frac{1}{16} >$

   (b) also plot the continuous empirical spectrum smoothed out by Parzen lag window with $M = 40$,

   (c) and find the kernel.which would achieve the same smoothing as the previous Parzen lag window (make sure the answer is properly simplified).

3. Assuming a sample of N equilibrated, consecutive observations from the following ARMA model

$$X_n + 0.9X_{n-1} = \varepsilon_n - 1.3\varepsilon_{n-1} - 0.9\varepsilon_{n-2}$$

   where the ε_n are IID from a Normal distribution with zero mean and $\sigma = 3.6$, find

   (a) the exact (no decimals) and properly simplified general formula for the corresponding ρ_k (don't forget to spell out 'exceptional' values, if any),

   (b) the asymptotic variance of each r_1 and r_2, and their covariance - in this part, you can switch to decimals.

4. Given that $V = 10$ and

$$\rho_k = 0.9^k + \frac{k}{10} \cdot (-0.5)^k$$

for all non-negative, integer values of k, find

(a) the corresponding exact (no decimals) spectral density as a ratio of two *trigonometric* polynomials (each a linear combination of $\cos\beta$, $\cos 2\beta$, $\cos 3\beta$, etc.); also plot the answer to establish whether this spectral density is 'legitimate',

(b) and all corresponding ARMA models (including the value of σ) - in this part, you can switch to decimals.