1. Find ML estimates of the six parameters (α_1, α_2, α_3, α_4, μ and σ) of the AR(4) model, based on the posted (equilibrated) data.

   Taking the estimates to be a good representation of the 'true' values of these parameters, find (and plot) the corresponding 'theoretical' spectral density (using $\cos\beta$, $\cos 2\beta$, $\cos 3\beta$ and $\cos 4\beta$) and the value of the common variance V.

2. Plot the 'continuous' *empirical* spectral density of the data from Question 1, smoothed out

   (a) by using the following kernel

   $$g(u) = \begin{cases} 25u + 5 & -\frac{1}{5} \le u \le 0 \\ 5 - 25u & 0 \le u \le \frac{1}{5} \\ 0 & \text{otherwise} \end{cases}$$

   (b) and also by using the following lag window

   $$\lambda_k = \begin{cases} 1 & k \le 20 \\ \frac{1}{2} - 3 \cdot \dfrac{k-30}{40} + \dfrac{(k-30)^3}{4000} & 20 \le k \le 40 \\ 0 & \text{otherwise} \end{cases}$$

3. Using the provided equilibrated data generated from an ARMA model (with $\mu = 0$), find the 'primitive' estimates of V, ρ_1, ρ_2 and ρ_3. Based on these, estimate the partial correlation coefficient between X_n and X_{n+3}, given the values of X_{n+1} and X_{n+2}.

4. Assuming that Z_1, Z_2, Z_3, Z_4 and Z_5 is a random independent sample of size 5 from $\mathcal{N}(0,1)$, and that

   $$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{bmatrix} = \begin{bmatrix} 0 & 3 & 2 & -1 & 4 \\ -3 & -1 & 0 & 1 & 3 \\ -2 & 1 & -4 & 4 & -2 \\ -3 & 5 & -1 & 2 & 0 \\ 2 & 3 & 0 & 0 & -5 \end{bmatrix} \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{bmatrix} + \begin{bmatrix} 3 \\ -2 \\ 4 \\ -1 \\ 2 \end{bmatrix}$$

   find the variance-covariance matrix of the five X's. Also compute

   (a) $\Pr\left(X_3 > 4.4 \mid X_1 = 1.8 \cap X_2 = -1.8 \cap X_5 = 2.3\right)$,

   (b) $\Pr\left(X_3 - X_4 > 2 \mid X_1 = 1.8 \cap X_5 = 2.3\right)$,

   (c) $\mathbb{E}\left(X_3^2 \cdot X_4^2\right)$.

5. Consider the following (theoretical) spectral-density function:

   $$w(\beta) = c \cdot \frac{1134 - 1296\cos(\beta) + 486\cos(2\beta)}{241 - 312\cos(\beta) + 72\cos(2\beta)}$$

   Find

(a) the corresponding ARMA models (all distinct ones),

(b) for each, the value of $\dfrac{\sigma^2}{V}$ where V is the common variance (after equilibration),

(c) a general (and exact) formula for ρ_k. Partial mark given for a list of ρ_1, $\rho_2, \dots \rho_{10}$.