Up to now, it was kind of silly to struggle with approximations to distributions having exact answers. Our last few questions do it for distributions nobody can ever find exactly.

1. Find the $\frac{1}{n^3}$ proportional term of the fourth cumulant of $g(\bar{X})$.
2. Consider a RIS of size n from a distribution with the following PDF

$$f(x) = \begin{cases} \sin(x) & 0 < x < \frac{\pi}{2} \\ 0 & \text{otherwise} \end{cases}$$

   Find (and spell out fully, including the support, in your report, using the 'where $z = ...$' style) an $\frac{1}{n}$-accurate approximation to the PDF of

   (a)
   $$Y = \overline{\ln X - X^2}$$

   (b)
   $$U = \ln \bar{X} - \bar{X}^2$$

   (c)
   $$V = \bar{X}^\alpha$$

   where α is chosen to remove (to the $\frac{1}{n}$ accuracy) the corresponding skewness.

3. Consider a RIS of size n from a bivariate distribution with the following joint PDF

$$f(x, y) = \begin{cases} 2x + 4y & 0 < x + y < 1 \\ 0 & \text{otherwise} \end{cases}$$

   Find an $\frac{1}{n}$-accurate approximation for the PDF of

$$\ln\left(\overline{X^2 \cdot \sin Y}\right)$$