

1. Design a transformation of two independent uniform RVs U_1 and U_2 into two RVs with the following bivariate PDF

$$f(x, y) = \begin{cases} c \cdot \frac{1 - xy}{1 - x} & \text{when } x > 0, y > 0 \text{ and } x + y < 1 \\ 0 & \text{otherwise} \end{cases}$$

where c is an appropriate constant.

2. Consider a tri-variate distribution whose PDF is proportional to

$$\exp(z - x)$$

when $x > 0, y > 0, z > 0$ and $x + y + z < 1$ (equal to 0 otherwise). Design a (numerical, if necessary) algorithm for transforming three independent $\mathcal{U}(0, 1)$ -type RVs into (X, Y, Z) . Based on the corresponding Maple code, generate a RIS of size 1000 from this distribution to estimate ρ_{xy} (including the corresponding standard error); test whether your estimate agrees with the actual value of ρ_{xy} (for a large sample size n , the r_{xy} estimator is approximately Normal with the mean of ρ_{xy} and standard deviation of $\frac{1}{\sqrt{n}}$).