

1. Consider the *multiplicative* group of integers mod 170,859,375. Which additive group (of the \oplus type) is this group isomorphic to? What is the highest possible order of an element of this group? How many such elements (of the highest order) are there, and what set of conditions must they meet (make sure to get your ANDs and ORs straight).
2. Which elements (in terms of the conditions they must meet) of the multiplicative group mod 29^k (where $k > 1$) have the order of $4 \times 29^{k-1}$, $7 \times 29^{k-1}$, $14 \times 29^{k-1}$, and $28 \times 29^{k-1}$ (these are four distinct questions).
3. Design a uniform (between 0 and 1) random-number generator based on

$$x_{n+1} = a \cdot x_n \quad \text{mod } 582,476,316,583$$

How many possible choices of a are there to get the longest possible sequence, and how long is such a sequence? Find one such a (make it 6 digits long) and verify, computationally, that it does yield the longest sequence. How do you choose x_0 ?

4. (Continuation of the previous question). Generate 10,000 such numbers and test (using Kolmogorov-Smirnov) whether they are uniformly distributed. Similarly test whether the corresponding first-order serial correlation coefficient is equal to zero.