1. Show how to transform U_1 and U_2 (independent, uniform between 0 and 1) into X and Y with the following joint PDF:

$$f(x,y) = \frac{x}{y^2} \cdot \exp(1 - \frac{x}{y}) \quad \text{when} \quad 0 < y < 1 \ \text{ and } \ y < x < 1 \qquad (0 \text{ otherwise})$$

2. Assuming a RIS of size n from the bivariate distribution of the previous question, find the following joint central moment

$$\mathbb{E}\left((\bar{X} - \mu_x)^3 \cdot (\bar{Y} - \mu_y)^2\right)$$

expanded in powers of $\frac{1}{n}$ (please give the coefficients of the expansion in decimal - that goes for the next two questions as well). Also spell out the (decimal) values of $\mu_x \equiv \mathbb{E}(X)$ and $\mu_y \equiv \mathbb{E}(Y)$.

3. Assuming a RIS of size n from the bivariate distribution of Question 1, find the $\frac{1}{n}$-accurate approximation to the PDF of $\sin\left(\overline{X+Y}\right)$.

4. Find the normal equation for the MLE of σ (assumed to be positive) based on a RIS of size n from a distribution whose PDF is

$$f(x) = \frac{1}{(1 + \frac{x}{\sigma})^{1+\sigma}} \qquad \text{when } x > 0 \qquad (0 \text{ otherwise})$$

We know that the estimator can be expanded in the following manner

$$\hat{\sigma} = \sigma + \overline{U} + ...$$

Find U and its variance. Spell out the Normal approximation for the PDF of $\hat{\sigma}$.

Assuming that this is your RIS: 0.35, 0.45, 3.93, 0.041, 0.097, 24.32, 4.35, 0.83, 1.81, 11.60, find the value of $\hat{\sigma}$.

5. What set of conditions must a meet to generate the longest possible sequence using

$$x_n = a \cdot x_{n-1} \qquad \text{mod } 2213314919066161$$

where $x_0 = 1$? What is the length of such a sequence? How many values of a are there which generate a sequence of this length? Find one such a (make it 8 digit long).