1. Show how to transform U_1 and U_2 (independent, uniform between 0 and 1) into X and Y with the following joint PDF:

   $f(x,y) = 2(y-x)\cdot\exp[-(y-x)^2]$ when $x < y < \infty$ and $0 < x < 1$ (0 otherwise)

2. Assuming a RIS of size n from the bivariate distribution of the previous question, find the following joint central moment

$$\mathbb{E}\left((\overline{X^2} - \mu_{x^2})^2 \cdot (\bar{Y} - \mu_y)^3\right)$$

   expanded in powers of $\frac{1}{n}$ (please give the coefficients of the expansion in decimal - that goes for the next two questions as well). Also spell out the (decimal) values of $\mu_{x^2} \equiv \mathbb{E}(X^2)$ and $\mu_y \equiv \mathbb{E}(Y)$.

3. Assuming a RIS of size n from the bivariate distribution of Question 1, find the $\frac{1}{n}$-accurate approximation to the PDF of

$$U \equiv \frac{n}{\sum_{i=1}^{n} \frac{X_i}{Y_i}}$$

   Use this approximation and $n = 15$ to evaluate $\Pr(2.5 < U < 3)$. Note that U is a simple function of a sample mean.

4. Assuming a RIS of size n from the bivariate distribution of Question 1, find the Normal approximation to the *joint* PDF of

$$V \equiv \frac{\bar{Y} - \overline{X^2}}{\bar{Y} + \overline{X^2}}$$

   and

$$W \equiv \exp\left(\bar{Y} - \overline{X^2}\right)$$

   Use this approximation and $n = 30$ to evaluate $\Pr(V < 0.6 \cap W > 2.8)$.

5. Consider the multiplicative group of integers mod 8649755859375. Which additive group (of the $\oplus$ type) is this group isomorphic to? How many of its elements are of order 759375, and what set of conditions do they meet? Find one such element (make it 6 digits long) and demonstrate numerically that it has the correct order.