1. Consider a random independent sample of n pairs of observations from a distribution with the following bivariate pdf:

$$f(x,y) = \begin{array}{cc} 2x(x-y) & \text{for} \left\{ \begin{array}{c} 0 < x < 1 \\ -x < y < x \end{array} \right. \\ 0 & \text{otherwise} \end{array}$$

Compute

$$\mathbb{E}\left[\left(\bar{X}-\mu_x\right)^2\left(\bar{Y}-\mu_y\right)^2\right]$$

2. Derive the ML estimator (let us call it B) of the β parameter of the following Weibull distribution, whose pdf is given by

$$f(x) = \begin{array}{cc} \frac{2x}{\beta^2}\exp(-\frac{x^2}{\beta^2}) & x > 0 \\ 0 & \text{otherwise} \end{array}$$

where $\beta > 0$.

(a) Find the approximate (Normal-based) distribution of B, to the $\frac{1}{n}$ accuracy.

(b) Plot this distribution for $n = 15$ and $\beta = 2$.

(c) Using Maple's uniform distribution, generate 1000 random independent samples of size 15 from the above Weibull distribution with $\beta = 2$, and compute B for each of these samples.

(d) Based on the results of Part c, construct and plot an approximate pdf of B (expecting to get a similar result to that of Part b).

3. Consider the multiplicative group mod 1005010010005001. Check whether 888888888888888 is one of the elements of this group, and if the answer is yes, establish its order.

4. Using Metropolis sampling, step size of 0.5, an ensemble of size 100, and 100 equilibrated iterations, estimate the values of

$$\frac{\displaystyle\iint \frac{|\mathbf{r}_1-\mathbf{r}_2|\cdot e^{-r_1-r_2}}{1+r_1+r_2}d\mathbf{r}_1 d\mathbf{r}_2}{\displaystyle\iint \frac{e^{-r_1-r_2}}{1+r_1+r_2}d\mathbf{r}_1 d\mathbf{r}_2}$$

and

$$\frac{\displaystyle\iint \frac{1}{|\mathbf{r}_1-\mathbf{r}_2|}\cdot\frac{e^{-r_1-r_2}}{1+r_1+r_2}d\mathbf{r}_1 d\mathbf{r}_2}{\displaystyle\iint \frac{e^{-r_1-r_2}}{1+r_1+r_2}d\mathbf{r}_1 d\mathbf{r}_2}$$

where the integration is over all 6-dimensional space, and $r_1 \equiv |\mathbf{r}_1|$ and $r_2 \equiv |\mathbf{r}_2|$.