1. Show how to transform $U_1$ and $U_2$ (independent, uniform between 0 and 1) into $X$ and $Y$ with the following joint PDF:

$$f(x,y) = \frac{\sin x \cdot \sin y}{2} \quad \text{when} \quad x > 0,\ y > 0, .x + y < \pi \qquad (0 \text{ otherwise})$$

2. Assuming a RIS of size $n$ from the $X$-$Y$ distribution of the previous question, find the following joint central moment

$$\mathbb{E}\left((\bar{X} - \frac{3\pi}{8})^4(\bar{Y} - \frac{3\pi}{8})^2\right)$$

expanded in powers of $\frac{1}{n}$ (please give the coefficients of the expansion in decimals - that goes for the next 2 questions as well).

3. Assuming a RIS of size $n$ from the $X$-$Y$ distribution of Question 1, find the $\frac{1}{n}$-accurate approximation to the PDF of $\ln\left(\overline{X+Y}\right)$. Clearly spell out (in decimal) the first 4 cumulants of $\ln\left(\overline{X+Y}\right)$.

4. Assuming a RIS of size $n$ from the $X$-$Y$ distribution of Question 1, find the basic Normal approximation to the bivariate PDF of $\overline{\ln(X+Y)}$ and $\overline{\ln|X-Y|}$. Spell out the two means, two variances and covariance.

5. Assuming that $a = 14681251$ and $x_0 = 984572$, what is the length of the sequence generated by

$$x_n = a \cdot x_{n-1} \qquad \bmod 12762815625$$

How many values of $a$ are there which would generate a sequence of this length?

What set of conditions must these $a$'s meet (hint: think in terms of the multiplicative subgroups)? Verify that 14681251 does.