1. Find a $\frac{1}{\sqrt{n}}$-accurate pdf of the ML estimator of α (let's call it $\hat{\alpha}$) based on a RIS of size n from the $\mathsf{gamma}(\alpha, 1)$ distribution. Using this approximation and $\alpha = 0.3$, $n = 15$, estimate $\Pr(0.25 < \hat{\alpha} < 0.35)$.

2. Compute the skewness (to the $\frac{1}{\sqrt{n}}$ accuracy) of $g(\bar{X})$, where $\bar{X}$ is a sample mean of a RIS of size n from a distribution with the following pdf

$$f(x) = \begin{cases} \dfrac{2x}{a^2} & 0 < x < a \\ 0 & \text{otherwise} \end{cases}$$

(where $a > 0$) and $g(..)$ is an arbitrary function. Find $g(..)$ to make this skewness equal to 0, and construct an $\frac{1}{\sqrt{n}}$-accurate pdf of this $g(\bar{X})$. Using this approximation and $a = 0.3$, $n = 15$, estimate $\Pr(0.17 < \bar{X} < 0.23)$.

3. Suppose X and Y are the number of spades and clubs (respectively), when dealing 5 cards from the ordinary deck. Assume that this random experiment is repeated, independently, 25 times. Find an $\frac{1}{\sqrt{n}}$ accurate approximation for the (discrete) joint probability function of $\sum_{i=1}^{n} X_i$ and $\sum_{i=1}^{n} Y_i$. Using this approximation, estimate $\Pr\left(\sum_{i=1}^{n} X_i = 31 \cap \sum_{i=1}^{n} Y_i = 35\right)$.

4. Generate 100 ensambles of size 200 (don't print them) from a 3D distribution whose joint pdf is proportional to

$$\frac{1}{1 + x^4 + y^6 + z^8}$$

for any real x, y and z. Use the results of this simulation to estimate $\mathbb{E}\left(\frac{1}{1+X^4+Y^6+Z^8}\right)$. What step size did you use, and why? How many iterations did you need to equilibrate?

5. Find a and x_0 (having five digits each) such that the sequence generated by

$$x_n = a \cdot x_{n-1} \qquad \mod 726572699$$

has the longest possible cycle. Using 200 consecutive numbers of this sequence, generate and plot a RIS of size 100 from a bivariate distribution with the following joint pdf

$$\begin{cases} 2e^{-x-y} & 0 < x < y \\ 0 & \text{otherwise} \end{cases}$$