Enter all (complete, fully simplified, and properly presented) answers in your booklet - do NOT refer to Maple (but DO email it to jvrbik@brocku.ca).

1. The random variables X, Y and Z have the following tri-variate PDF

$$2x \cdot \frac{z^{x-1}}{y^x} \cdot \exp\left(-x^2 - \frac{y}{x}\right) \qquad \text{when } 0 < x,\ 0 < y \ \text{ and } \ 0 < z < y$$

(zero otherwise). Express X, Y and Z as a transformation of three independent RVs U_1, U_2 and U_3, each uniformly distributed over the $(0,1)$ interval (fully *simplify* each of your answers!). Hint: start with the X marginal, then $Y|X$ etc.; in your integrals, don't forget to 'assume' that $x > 0$.

2. How many elements of the multiplicative group mod $219,503,494,144$ have the order of 22 ? Provide one sufficient set of conditions for an element of this group to be of that order and find one such element. What is the *highest* possible order of an element of the same group, how many such elements are there, and what set of conditions (sufficient and necessary) must they meet? Find one such element (make it 6 digits long), select a 12 digit seed (the value of x_0) and, using these, generate a sequence of 4000 random numbers from $\mathcal{U}(0,1)$. Converting these into a sequence of independent random numbers from the exponential distribution with the mean of 3, test (by Kolmogorov-Smirnov) whether the last sequence is indeed a RIS from $\mathcal{E}(3)$.

3. Find the $\frac{1}{n}$-accurate approximation for the PDF of $\bar{X}$ when sampling from a distribution with the following PDF

$$f(x) = 3x^2 \exp\left(-x^3\right) \qquad \text{when } x > 0 \qquad \text{(zero otherwise)}$$

Repeat with $\bar{X}^\alpha$, where α is an exponent which removes the skewness of $\bar{X}^\alpha$ (convert to and quote the approximate PDF of $\bar{X}$). Use each of the two approximations and $n = 3$ to estimate $\Pr(\bar{X} > 0.9)$.

4. Consider the following bivariate PDF

$$f(x, y) = 2\exp\left(-x^2 - \frac{y}{x}\right) \qquad \text{when} \qquad x > 0 \text{ and } y > 0$$

(zero otherwise). Assuming a RIS of size n from this distribution, derive a formula for

$$\mathbb{E}\left((\overline{\ln Y} - \mu_{\ln y})^6\right) \quad \text{ and for } \quad \mathbb{E}\left((\overline{X} - \mu_x)^2 \cdot (\overline{\ln Y} - \mu_{\ln y}) \cdot (\overline{X^2} - \mu_{x^2}) \cdot (\overline{Y^2} - \mu_{y^2})\right)$$

(spell out the means as well).

5. Continuation of the previous question: Find the basic Normal approximation for the bivariate distribution of

$$U = \sqrt{\overline{X^2}} \cdot \exp\left(\overline{\ln Y}\right)$$

and

$$V = \frac{1 + (\ln \bar{X})^2}{\overline{Y^2}}$$

(just spell out the values of the usual 5 parameters: the 2 means, the 2 variances and the correlation coefficient).