

**Lab #5: PALEOHYDRAULIC PROBLEMS**

Solve each of the following problems with the information provided (additional information which may be useful in each problem is given in table I). In each case, list any/all assumptions that were made in the technique to solve each problem.

**Note:** before attempting these problems refer to the example given on pages 2 to 3 of this laboratory exercise.

1. In outcrop, the erosional base of a straight fluvial channel ascends from an elevation of 189.4 metres to 189.6 metres over a horizontal distance of 227 metres. A thin bed of quartz-pebble conglomerate lies on the erosional base of the channel. The conglomerate is very well-sorted, well-rounded, has high sphericity, and has an average size (diameter) of 8 mm. What was the flow depth in the channel when the gravel was deposited?
2. A placer gold deposit contains gold particles with an average size of  $4.8\phi$  while the average size of associated quartz pebbles is  $-3.2\phi$ . Was the gold transported with the gravel as bed-load or did it filter into the gravel from suspension?
3. Several dangerous chemicals are discharged from an industrial source into a river in the Niagara Peninsula. The chemicals are transported along the river in solution and adsorbed onto clay suspended in the flow (the average size of the clay is  $8.6\phi$ ). From the site of discharge into the river to the its mouth at lake Erie the river descends at a rate of .1 m per km and has an average depth of 5 m and a width of 100 m. Is the discharge in the river adequate to ensure that most of the adsorbed toxins will be transported all the way to the lake? If you have used a very simplistic approach to solve this problem briefly discuss some complications that you would expect.

**Table I. Additional Data**

Sediment density (assume quartz if not known) =  $2650 \text{ kg/m}^3$

Density of gold =  $19,300 \text{ kg/m}^3$

Fluid density (assume water at 20 degrees C) =  $998.2 \text{ kg/m}^3$

Dynamic viscosity (assume water at 20 degrees C) =  $1.005 \times 10^{-3} \text{ N}\cdot\text{s/m}^2$

Acceleration due to gravity  $9.806 \text{ m/s}^2$

### Example problem

An ancient channel, enclosed in shale and filled with very fine sand is 2 m deep (Fig. 1). The floor of the channel is mantled by a layer of 1 cm diameter, spherical pebbles. (1) What was the average flow velocity in the channel? (2) Was the sand transported as bedload or suspended load?

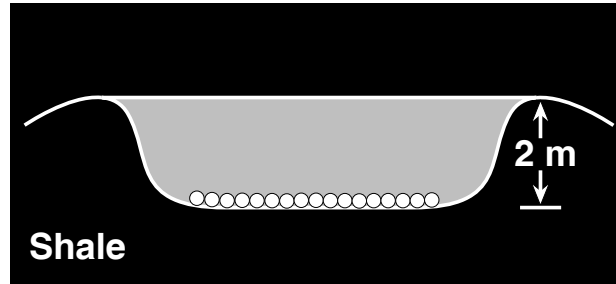


Figure 1.

Question 1 may be determined by using the law of the wall for rough turbulent boundaries:

$$\frac{u}{U_*} = 8.5 + \frac{2.3}{\kappa} \log \frac{y}{y_o} \quad \text{Eq. 1}$$

where  $u$  is the velocity at height  $y$  above the boundary,  $k=0.4$  and  $y_o=d/30$  ( $d$  is the size of the grains on the boundary; i.e., 0.01m). Average velocity occurs at  $y=0.4D$  ( $D$  is depth of flow). Rearranging, substituting and solving for  $u$  at  $y=0.4D$ :

$$u = \left( 8.5 + \frac{2.3}{0.4} \log \frac{0.4 \times 2}{0.01 \div 30} \right) U_* = 27.94 U_* \quad \text{Eq. 2}$$

Thus, we have a general relationship for solving for  $u$  but we need to determine the shear velocity ( $U_*$ ) where:

$$U_* = \sqrt{\frac{\tau_o}{\rho}} \quad \text{Eq. 3}$$

where  $\rho$  is the fluid density and  $\tau_o$  is the boundary shear stress. The boundary shear stress can be determined using Shields' curve if we assume that the 0.01 m diameter spheres are the largest particles that could be moved by the current. This boundary shear stress can, in turn, be used to calculate  $U_*$ . To determine the boundary shear stress calculate the value of the quantity:

$$\frac{d}{\nu} \sqrt{0.1 \times \left( \frac{\rho_s}{\rho} - 1 \right) \times 9.806 \times d} \quad \text{Eq. 4}$$

where  $\rho_s=2650 \text{ kg/m}^3$  (assuming that the pebbles have the density of quartz),  $\rho=998.2 \text{ kg/m}^3$

(assuming that the fluid was water at 20°C) and  $d$  is 0.01 m. The kinematic viscosity ( $\nu$ ) can be determined from the dynamic viscosity ( $\mu=1.005 \times 10^{-3} \text{Ns/m}^2$ ) given the relationship:

$$\nu = \mu / \rho \quad \text{Eq. 5}$$

Thus, the kinematic viscosity is  $1.007 \times 10^{-6}$  (water at 20°C) and the value of Eq. 4 is 1265. Locating this value on Shields' curve indicates a value of Shields  $\beta=0.06$ , where

$$\beta = \frac{\tau_o}{(\rho_s - \rho)gd} \quad \text{Eq. 5}$$

( $g=9.806 \text{ m/s}^2$ ). By rearranging and substituting the appropriate values we can determine the boundary shear stress for the flow that transported the pebbles:

$$\tau_o = \beta(\rho_s - \rho)gd = 0.06(2650 - 998.2) \times 9.806 \times 0.01 = 9.72 \text{ Nm}^{-2} \quad \text{Eq. 6}$$

Substituting this value for the boundary shear stress in Eq. 3 and solving yields  $U_* = 0.099 \text{ m/s}$ .

Substituting the shear velocity into Eq. 2 and solving for  $u$  yields a **mean flow velocity of 2.77 m/s**.

Now, the second question, was the very fine sand filling the channel in suspension or could it move as bedload with the gravel?

Middleton's criterion for suspension suggests that if the shear velocity exceeds the settling velocity of the very fine sand than the sand must have been transported in suspension. We can calculate the settling velocity of very fine sand (the upper limit for this grade is 0.125 mm) using Stokes' Law of Settling:

$$\omega = \frac{(\rho_s - \rho)gd^2}{18\mu} \quad \text{Eq. 7}$$

Assuming that the sand grains have the density of quartz and the water was 20°C we can use the values given above and will find that  $\omega=0.014 \text{ m/s}$  for  $d=0.000125 \text{ m}$ . This is clearly less than the shear velocity, therefore the **sand was transported in suspension**.