The normalizer of a metabelian group in its integral group ring

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In this paper, we discuss two situations for which all C-automorphisms are inner; therefore, the normalizer property holds for those cases. Our results generalize a result of Marciniak and Roggenkamp ([8], Theorem 12.3). As an application of our theorems, we prove that the normalizer property holds for the integral group ring of a split finite metabelian group with a dihedral Sylow 2-subgroup. Our results rely heavily on the methods of Marciniak and Roggenkamp in [8].

1 Introduction

Let G be a group and $\mathcal{U}(\mathbb{Z}G)$ be the group of units of the integral group ring $\mathbb{Z}G$. It is a classical problem in the theory of group rings to investigate the normalizer $N_{\mathcal{U}}(G)$ of G in $\mathcal{U}(\mathbb{Z}G)$ (see [6, 13] for detail). It is clear that $N_{\mathcal{U}}(G)$ contains G and also contains $\mathcal{Z} = \mathcal{Z}(\mathcal{U}(\mathbb{Z}G))$, the subgroup of central units of $\mathcal{U}$.

Problem 43 in [13] asks whether $N_{\mathcal{U}}(G) = G\mathcal{Z}$ when G is finite. The equality was first shown to hold for finite nilpotent groups by Coleman [1],

*This research was supported in part by an NSERC grant. Accepted by Journal of Algebra, Sep. 13, 2000

and later extended by Jackowski and Marciniak [5] to all finite groups having a normal Sylow 2-subgroup. In particular, this property holds for all finite groups of odd order. Mazur perceived that there is a close relation between this question and the isomorphism problem (see Mazur [9, 10, 11]). Hertweck first found counterexamples to the normalizer problem, and then, using them and a clever generalization of Mazur's results, he managed to construct a counterexample to the isomorphism problem ([2, 3, 4]). Because of the connection to the isomorphism problem, it is still of interest to know which groups enjoy the normalizer property.

Recently, Parmenter, Sehgal and the author [7] proved that the normalizer property holds for any finite group G such that $R(G)$ is not trivial, where $R(G)$ denotes the intersection of all nonnormal subgroups of G. In the meanwhile, Marciniak and Roggenkamp [8] proved that this property holds for finite metabelian groups with an abelian Sylow 2-subgroup. In this paper, we discuss two important situations for which all C-automorphisms are inner; therefore, the normalizer property holds for those cases (Theorems 2.8 and 2.17). Using these, we extend the above mentioned result of Marciniak and Roggenkamp to some metabelian groups with not necessarily abelian Sylow 2-subgroups. For example, we prove that the normalizer property holds for the integral group ring of a split finite metabelian group with a dihedral Sylow 2-subgroup. Our results rely heavily on the methods of Marciniak and Roggenkamp in [8].

2 The normalizer $N_{\mathcal{U}}(G)$ for metabelian groups

Any unit $u \in N_{\mathcal{U}}(G)$ determines an automorphism $\rho = \rho_u$ of G such that $\rho(g) = ugu^{-1}$ for all $g \in G$. We now consider the subgroup $Aut_{\mathcal{U}}(G)$ formed by all such automorphisms and it is not hard to see that the normalizer problem described in [13] is equivalent to Question 3.7 in Jackowski and Marciniak [5]:

“ Is $Aut_{\mathcal{U}}(G) = Inn(G)$ for all finite groups?”

It is convenient to use this equivalent form to discuss the normalizer problem here, and we will describe some finite metabelian groups for which the normalizer property holds.

Every automorphism in $Aut_{\mathcal{U}}(G)$ automatically satisfies several properties described by Coleman and the following definition was introduced by Marciniak and Roggenkamp in [8].

Definition 2.1. *An automorphism ρ of a finite group G is called a Coleman automorphism, or a C-automorphism for short, if $\rho^2 = \rho \circ \rho$ is inner, ρ preserves the conjugacy classes in G and for every Sylow p-subgroup P of G, we have $\rho|_P = conj(g)|_P$ for some $g \in G$.*

Remark 2.2. *Our interest in those automorphisms comes from the fact that all automorphisms in $Aut_{\mathcal{U}}(G)$ are C-automorphisms. Therefore, if all C-automorphisms of G are inner, then the normalizer property holds for G.*

We first describe a necessary and sufficient condition for a C-automorphism of a metabelian group to be inner.

Proposition 2.3. *Let G be a metabelian group and let B be an abelian normal subgroup of G for which the quotient group $A = G/B$ is also abelian. Then a C- automorphism ρ of G is inner if and only if $\rho|_{B\cup P} = conj(g)|_{B\cup P}$ for some $g \in G$, where P is a Sylow 2-subgroup of G.*

The proof is standard, but we include it for completeness.

Proof. One direction is clear. Let ρ be a C-automorphism of G such that $\rho|_{B\cup P} = conj(g)|_{B\cup P}$. Substituting for ρ a new C-automorphism $\rho \circ conj(g^{-1})$, we have $\rho|_B = id|_B$ and $\rho|_P = id|_P$. Note that the subgroup generated by B and P is exactly the preimage H of A_2 in G; i.e. $H = \langle B, P\rangle$. Thus $\rho|_H = id|_H(*)$. Since ρ preserves conjugacy classes, obviously it induces the identity on A. Now we define a map $\delta : A \longrightarrow B$ by $\delta(a) = \rho(y)y^{-1}$ for all $a \in A$, where y is any preimage of a in G. It is routine to check that δ is well defined, $\delta(a) \in B$, and δ is a 1-cocycle. Thus $[\delta] \in H^1(A, B)$. Note that $H^1(A, B) = H^1(A_2 \times A_2', B) = H^1(A_2, B) \times H^1(A_2', B)$. We first note that the restriction of $[\delta]$ to A_2 is trivial. This is because for all $a_2 \in A_2$, $\delta(a_2) = \rho(h)h^{-1} = 1$, where h is any preimage of a_2 , so $\rho(h) = h$ by assumption $(*)$. Next let $k = |A_2'|$. It is well known that the restriction of $[\delta^k]$ to A_2' is trivial. Thus $[\delta^k]$ is trivial and then δ^k is a coboundary. Therefore, ρ^k is inner. Since ρ^2 is inner and k is odd, it follows that ρ is inner. □

Remark 2.4. *Proposition 2.3 states that a C-automorphism ρ is inner if and only if the restriction of ρ to the preimage H of A_2 in G is inner (i.e. ρ acts on H as conjugation by a group element).*

Proposition 2.3 can be easily rephrased to give the following.

Proposition 2.5. *Let G be a metabelian group and let B be an abelian normal subgroup of G for which the quotient group $A = G/B$ is also abelian. Then the normalizer property holds for G if and only if for every $\rho \in Aut_{\mathcal{U}}(G)$, $\rho|_{B \cup P} = conj(g)|_{B \cup P}$ for some $g \in G$, where P is a Sylow 2-subgroup of G.*

Corollary 2.6. *Let G be a metabelian group and let B be an abelian normal subgroup of G for which the quotient group $G/B = A$ is abelian. If P is a Sylow 2-subgroup of G, and $P = B_2 \rtimes D$, where B_2 is the Sylow 2-subgroup of B and D is a 2-subgroup of G, then a C-automorphism ρ is inner if and only if $\rho|_B = conj(g)|_B$ for some $g \in G$.*

Proof. The necessity is obvious.

Conversely, by multiplying by a conjugation of a group element, we may assume that $\rho|_B = id|_B$. Since ρ is a C-automorphism, $\rho|_P = conj(h)|_P$. Since ρ^2 is inner, if some odd power of ρ is inner, then ρ is inner. By taking some suitable odd power of ρ, we may assume that the order of ρ is a power of 2 and h is a 2-element. Note that ρ permutes the set of all Sylow 2-subgroups of G. As the order of ρ is a power of 2, and the number of permuted groups is odd, this permutation action has a fixed point. That is to say there exists a Sylow 2-subgroup P' of G such that $\rho(P') = P'$. Without loss of generality, we may assume that $P' = P$. We note that D is abelian since every commutator $[d, d_1]$ in D has image the identity of A and thus $[d, d_1] \in D \bigcap B_2 = 1$. $\rho(P) = conj(h)(P) = P$ and thus h is in the normalizer of P in G. As h is a 2-element, it follows that h belongs to P. Now we write $h = b_2 d$ where $b_2 \in B_2$ and $d \in D$. Since $id|_{B_2} = \rho|_{B_2} = conj(b_2 d)|_{B_2} = conj(d)|_{B_2}$, we have that $d \in C(B_2)$. Moreover, as D is abelian, it follows that $d \in C(P)$. Let $\rho_1 = conj(b_2^{-1}) \circ \rho$. Then $\rho_1|_B = conj(b_2^{-1}) \circ \rho|_B = conj(b_2^{-1})|_B = id|_B$ and $\rho_1|_P = conj(d)|_P = id|_P$. It follows from Proposition 2.3 that ρ_1 is inner. Therefore ρ is inner and we are done. □

Remark 2.7. *G is called a split metabelian group if G is a semidirect product of an abelian normal subgroup B of G by an abelian subgroup A of G, i.e. $G = B \rtimes A$. We note that if G is a split metabelian group, then the assumption on a Sylow 2-subgroup of G in Corollary 2.6 is automatically satisfied.*

We now describe a family of finite metabelian groups for which the normalizer property holds.

Theorem 2.8. *Let G be a finite metabelian group and let B be an abelian normal subgroup of G for which the quotient group $G/B = A$ is abelian. If $P = B_2 \rtimes D$ and $C_{A_2}(B_2) = C_{A_2}(b_2)$ for some $b_2 \in B_2$, where P is a Sylow 2-subgroup of G, and B_2 and A_2 are the Sylow 2-subgroups of B and A respectively, then every C-automorphism of G is inner, and therefore the normalizer property holds for G.*

We prove Theorem 2.8 by means of the following two lemmas.

Lemma 2.9. *Let P be a finite abelian 2-group acting on a finite abelian group M of odd order. Then there is an element $m \in M$ such that $C_P(M) = C_P(m)$.*

Proof. M can be regarded as a P ($\mathbb{Z}P$)-module and thus M is a direct sum of indecomposable P-submodules M_i, i.e. $M = \sum M_i$. If for all submodules M_i, $C_P(M_i) = C_P(m_i)$ for some $m_i \in M_i$, then $C_P(M) = C_P(m)$ for $m = \sum m_i$. So it is enough to prove the lemma for M indecomposable. Let $Q = C_P(M)$. Then M is a faithful $R = P/Q$ -module. Suppose that the result is not true. Then for any nonzero element m of M, $C_P(M) \neq C_P(m)$. Thus there exists an element $p \in C_P(m)$, $p \notin C_P(M) = Q$, and then the image g of p in R is not the identity. Therefore, we have that $gm = m$ with $m \neq 0$ and $1 \neq g \in R$. Since g is a 2-element, by replacing g by the suitable power of g, we can assume that $g^2 = 1$. Note that multiplication by $1 - g$ is an endomorphism of the R-module M, but it is not an isomorphism. By the Fitting lemma ([12] 3.3.5, page 82) $1 - g$ is a nilpotent endomorphism of M. Note that $(1-g)^{2^k} = 2^{1+2+\cdots+2^{k-1}}(1-g)$. Since the order of M is odd, $1 - g$ annihilates M, i.e. $g \in C_R(M) = 1$. This contradiction finishes the proof. □

Remark 2.10. *Lemma 2.9 remains true for any finite abelian $p-$group P acting on a finite abelian group M of order prime to p.*

Lemma 2.11. *Let G be a finite metabelian group and let B be an abelian normal subgroup of G for which the quotient group $G/B = A$ is also abelian. Suppose that the Sylow 2-subgroup A_2 of A has the property that $C_{A_2}(B_2) = C_{A_2}(b_2)$ for some $b_2 \in B_2$ where B_2 is the Sylow 2-subgroup of B. Then any C-automorphism ρ of G acts as an inner automorphism of B, i.e. $\rho|_B = conj(g)|_B$ for some $g \in G$.*

Proof. B is a direct product of its Sylow subgroups B_p, which are normal in G. Let ρ be a C-automorphism of G. It acts on each B_p as an inner automorphism. As we mentioned earlier, by raising to a sufficiently high odd power, we can assume that ρ is of 2-power order. We can also assume that ρ acts on B_p as conjugation by a 2-element h_p, acts on a Sylow 2-subgroup P of G as conjugation by a 2-element h_2 and that ρ^2 is a conjugation by a 2-element. It is routine to check that ρ is a C-automorphism of the preimage H of A_2 in G (for example, it follows from Lemma 1 in [9] that ρ preserves conjugacy classes in H), so we can assume that $G = H$, i.e. $A = A_2$. Now there exist $b_p \in B_p$ such that $C_A(B_p) = C_A(b_p)$ by Lemma 2.9 and our assumptions. Let $b = \prod b_p$ be the product of the b_p's. Then there is $g \in G$ such that $gbg^{-1} = \rho(b)$. Hence $\prod gb_pg^{-1} = gbg^{-1} = \prod \rho(b_p) = \prod h_pb_ph_p^{-1}$ and thus $gb_pg^{-1} = h_pb_ph_p^{-1}$ for all p. It follows that $g^{-1}h_p$ centralizes b_p. Since this element acts on B_p as its image in A. we conclude that $g^{-1}h_p$ also centralizes B_P. Now for any element $b' = \prod b'_p \in B$, we have $\rho(b') = \prod \rho(b'_p) = \prod h_pb'_ph_p^{-1} = \prod h_p(h_p^{-1}g)b'_p(g^{-1}h_p)h_p^{-1} = \prod gb'_pg^{-1} = gb'g^{-1}$. This shows that ρ acts on B as conjugation by g. $\square$

Now Theorem 2.8 follows directly from Lemma 2.11 and Corollary 2.6.

Next we discuss several corollaries of Theorem 2.8 and Proposition 2.3. The first one is the result proved by Marciniak and Roggenkamp ([8] Theorem 12.3).

Corollary 2.12. *Let G be a finite metabelian group and let B be an abelian normal subgroup of G for which the quotient group $G/B = A$ is also abelian. If a Sylow 2-subgroup P of G is abelian, then every C-automorphism of G is inner.*

Proof. First notice that $C_{A_2}(B_2) = A_2 = C_{A_2}(b_2)$ for any $b_2 \in B_2$. It follows from Lemma 2.11 that for every C-automorphism ρ, $\rho|_B = conj(g)|_B$ for some $g \in G$. By conjugating by a group element, we may assume that $\rho|_B = id|_B$. As we remarked earlier, we may also assume that the order of ρ is a power of 2. Thus ρ fixes a Sylow 2-subgroup P of G. Now $\rho|_P = conj(h)|_P$. After taking a suitable odd power of ρ, we may assume that h is a 2-element, and therefore, h belongs to P. Since P is an abelian subgroup, we have $\rho|_P = id|_P$. The result follows from Proposition 2.3. $\square$

Corollary 2.13. *Let G be a finite metabelian group and let B be an abelian normal subgroup of G for which the quotient group $G/B = A$ is also abelian.*

If B_2, the Sylow 2-subgroup of B, is cyclic, then the restriction to B of every C-automorphism ρ of G is inner i.e. $\rho|_B = conj(g)|_B$ for some $g \in G$. In addition, if $P = B_2 \rtimes D$ where P is a Sylow 2-subgroup of G, then every C-automorphism of G is inner.

Corollary 2.14. *Let G be a finite metabelian group and let B be an abelian normal subgroup of G for which the quotient group $G/B = A$ is also abelian. If $P = C_{2^n} \rtimes C_2$, where P is a Sylow 2-subgroup of G and C_m is the cyclic group of order m, then the restriction to B of every C-automorphism ρ of G is inner i.e. $\rho|_B = conj(g)|_B$ for some $g \in G$.*

Proof. We need only show that $C_{A_2}(B_2) = C_{A_2}(b_2)$ for some $b_2 \in B_2$. Let $P = C_{2^n} \rtimes C_2 = \langle x \rangle \rtimes \langle y \rangle$. with $x^{2^n} = y^2 = 1, x^y = x^i$. We note that since B_2 is a subgroup of P, B_2 can be generated by at most two elements of P.

Case 1, If B_2 is cyclic, then it is obvious that $C_{A_2}(B_2) = C_{A_2}(b_2)$ for some $b_2 \in B_2$ being a generator of B_2.

Case 2, If $B_2 = \langle x^i, x^j y \rangle$, then in the quotient group $A_2, \bar{x}^j \bar{y} = 1$, so $\bar{y} = \bar{x}^{-j}$. It follows that $A_2 = \langle \bar{x}, \bar{y} \rangle = \langle \bar{x} \rangle$ is cyclic. Therefore A_2 commutes with x^i, and thus $C_{A_2}(B_2) = C_{A_2}(x^j y)$.

In any case, we have proved that $C_{A_2}(B_2) = C_{A_2}(b_2)$ for some $b_2 \in B_2$. The corollary follows from Lemma 2.11. □

The next result follows directly from Corollaries 2.6 and 2.14

Corollary 2.15. *Let $G = B \rtimes A$ be a split finite metabelian group, where B is an abelian normal subgroup and A is abelian. If a Sylow 2-subgroup P of G satisfies the condition $P = C_{2^n} \rtimes C_2$, then every C-automorphism of G is inner.*

Remark 2.16. *With the assumption of Corollary 2.15, in particular, if a Sylow 2-subgroup of G is a dihedral group, then every C-automorphism is inner. Therefore, the normalizer property holds for G.*

Next we discuss another situation for which the normalizer property holds

Theorem 2.17. *Let G be a finite metabelian group and let B be an abelian normal subgroup of G for which the quotient group $G/B = A$ is also abelian. If A_2, the Sylow 2-subgroup of A, is cyclic, then every C-automorphism of G is inner.*

Proof. We first show that $C_{A_2}(B_2) = C_{A_2}(b_2)$ for some $b_2 \in B_2$. Since A_2 is a cyclic 2-group, its subgroups are linearly ordered by inclusion. We can take b_2 to be any element $b \in B_2$ such that $C_{A_2}(b)$ is minimal. Now $C_{A_2}(b_2) = \cap C_{A_2}(b')(\forall b' \in B) = C_{A_2}(B_2)$. It follows from Lemma 2.11 that for every C-automorphism ρ of G, $\rho|_B = conj(g)|_B$ for some $g \in G$.

Next we show that there exists a Sylow 2-subgroup P of G such that $\rho|_P = conj(g)|_P$ for the same g as above. As we remarked earlier, without loss of generality, we may assume that $\rho|_B = id|_B$ and the order of ρ is a power of 2. Thus ρ fixes a Sylow 2-subgroup P of G and $\rho|_P = conj(h)|_P$. For the same reason mentioned before, we may assume that h is a 2-element and therefore, $h \in P$. It is clear that there is an element $g \in P$ such that $P = \langle B_2, g\rangle$ and g maps to a generator a of A_2. Write $h = b_2 g^i$ where $b_2 \in B_2$. As $\rho|_{B_2} = id|_{B_2}$, for all $b \in B_2$ we have $b = \rho(b) = conj(b_2 g^i)(b) = conj(g^i)(b)$. Thus $g^i \in C(B_2)$. Since g^i commutes with g, it follows that $g^i \in C(P)$. Let $\rho_1 = conj(b_2^{-1}) \circ \rho$. Then

$$\rho_1|_B = conj(b_2^{-1}) \circ \rho|_B = conj(b_2^{-1})|_B = id|_B$$

and

$$\rho_1|_P = conj(g^i)|_P = id|_P.$$

Applying Proposition 2.3 to our situation, we conclude that ρ is inner and this finishes the proof. □

The following corollary is the finite version of Theorem 2 in [7] which holds for several families of groups such as dihedral groups and Q^* groups.

Corollary 2.18. *Let $G = \langle H, g\rangle$ be a finite group, where H is an abelian subgroup of index 2. Then the normalizer property holds for G.*

We close by noting that Mazur [9] showed that there exist finite metabelian groups for which the normalizer property holds, but for which not all C-automorphisms are inner. Another example is as follows:

Remark 2.19. *In [8], Marciniak and Roggenkamp gave an example of a finite metabelian group of order 384 ($G \cong (C_2^4 \times C_3) \rtimes C_2^3$) for which there exists a C-automorphism which is not inner.*

Proposition 2.20. *Let G be a finite metabelian group and let B be an abelian normal subgroup of G for which the quotient group $G/B = A$ is also abelian. If $\mathcal{U}(\mathbb{Z}A)$ has only trivial units, then the normalizer property holds for G. In particular, it holds for the group $(C_2^4 \times C_3) \rtimes C_2^3$.*

Proof. Let $u \in N_{\mathcal{U}}(G)$. Then $u = \alpha_0 a_0 + \sum \alpha_i a_i$, where α_i are in $\mathbb{Z}B$, $a_0 = 1$, and all a_i form a right transversal to B in G.

In $\mathbb{Z}(G/B) \cong \mathbb{Z}A$, we have $\bar{u} = \epsilon(\alpha_0) + \sum \epsilon(\alpha_i)a_i$, where ϵ denotes the augmentation map. Since $\mathbb{Z}(G/B) \cong \mathbb{Z}A$ has only trivial units, only one $\epsilon(\alpha_l) = \pm 1$ and the other $\epsilon(\alpha_i) = 0$ for all $i \neq l$. Multiplying by $\pm a_l^{-1}$ if necessary, we can assume that $\epsilon(\alpha_0) = 1$ and $\epsilon(\alpha_i) = 0$ for all $i \neq 0$. Let ρ be the automorphism of G such that $\rho(g) = ugu^{-1}$ for all $g \in G$. We will show that $\rho|_{B \cup P} = conj(b_1)|_{B \cup P}$ for some $b_1 \in B$, where P is a Sylow 2-subgroup of G. Thus the result follows from Proposition 2.5. Since ρ preserves the conjugacy classes in G, it follows that $\rho(b) \in B$ for every $b \in B$. So there exists $b_0 \in B$ such that $b_0 b u = ub$. This implies that $b_0 b \alpha_0 = \alpha_0 b$. Therefore, $(b_0 - 1)\alpha_0 = 0$. This means that $\epsilon(\alpha_0)$ is divisible by the order of b_0, forcing $b_0 = 1$. Hence $[u, b] = 1$ for every $b \in B$ and then $\rho|_B = id|_B$. Next rewrite $u = \sum u(x)x$ where $u(x) \in \mathbb{Z}$ and $x \in G$. Since $u = \rho(g)ug^{-1}$, we have $\sum u(x)x = \sum u(x)\rho(g)xg^{-1}(*)$. Note that $\rho(g)xg^{-1} \in B$ provided that $x \in B$. We can define a group action σ_h of P on the set B as follows: $\sigma_h(x) = \rho(h)xh^{-1}$ for all $x \in B$. It follows from $(*)$ that $u(x)$ is a constant on each orbit of x. Since P is a 2-subgroup, every orbit of this action must have a 2-power length. Therefore, we have that

$$1 = \epsilon(\alpha_0) = \sum_{\forall x \in B} u(x) = \sum c_i 2^{l_i}$$

where 2^{l_i} is the length of the orbit of x_i and $u(x_i) = c_i$. This forces $p^{l_j} = 1$ for some j; that is to say there is a fixed point of this action, say $b_1 = x_j \in B$. Thus $\rho(h)b_1h^{-1} = b_1$ for all $h \in P$. Consequently, $\rho|_P = conj(b_1)|_P$ and then $\rho|_{B \cup P} = conj(b_1)|_{B \cup P}$. Therefore, ρ is inner and the normalizer property holds for G. □

Acknowledgements

The author wishes to express his thanks to Dr. S.K. Sehgal for many helpful and stimulating conversations. Special thanks are directed to the referee for providing to the author the statements and proofs of Lemma 2.9

and Remark 2.10, and for his other suggestions and advice in preparing the revised version of the manuscript.

REFERENCES

[1] D. B. Coleman, *On the modular group ring of a p-group*, Proc. Amer. Math. Soc. **15** (1964), 511–514.

[2] M. Hertweck, *A counterexample to the isomorphism problem for integral group rings of finite groups*, to appear in Comptes Rendu, Paris.

[3] ______, *Ganzzahlige Gruppenringe: Über den normalisator einer Gruppenbasis*, preprint 1996.

[4] ______, *A solution to the isomorphism problem*, preprint 1997.

[5] S. Jackowski and Z. Marciniak, *Group automorphisms inducing the identity map on cohomology*, J. Pure Appl. Algebra **44** (1987), no. 1-3, 241–250.

[6] W. Kimmerle,*On the normalizer problem*, Algebra: Some Recent Advances, Hindustan Book Agency, New Delhi, (1999), 89–98.

[7] Yuanlin Li, M.M.Parmenter, and S.K.Sehgal, *On the normalizer property for integral group rings*, Comm. Algebra **27** (1999), no. 9, 4217–4223.

[8] Z.S. Marciniak and K.W. Roggenkamp, *The normalizer of a finite group in its integral group ring and cech cohomology*, Accepted, 1998, J. Pure Appl. Algebra.

[9] M. Mazur, *Automorphisms of finite groups*, Comm. Algebra **22** (1994), 6259–6271.

[10] ______, *On the isomorphism problem for integral group rings of infinite groups*, Expo. Math. **13** (1995), 433–445.

[11] ______, *The normalizer of a group in the unit group of its group ring*, J. Algebra **212** (1999), no. 1, 175–189.

[12] Derek J.S. Robinson, *A course in the theory of groups*, Second edition, Springer-Verlag, New York 1996.

[13] S. K. Sehgal, *Units in integral group rings*, Longman Scientific & Technical Press, Harlow, 1993.